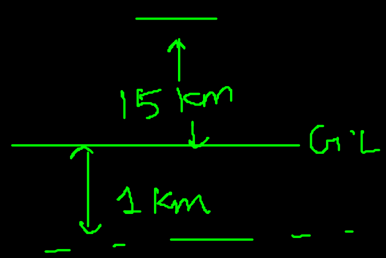
Hydrology-

⊛ Definition:- At a basic level, hydrology is the science of water.

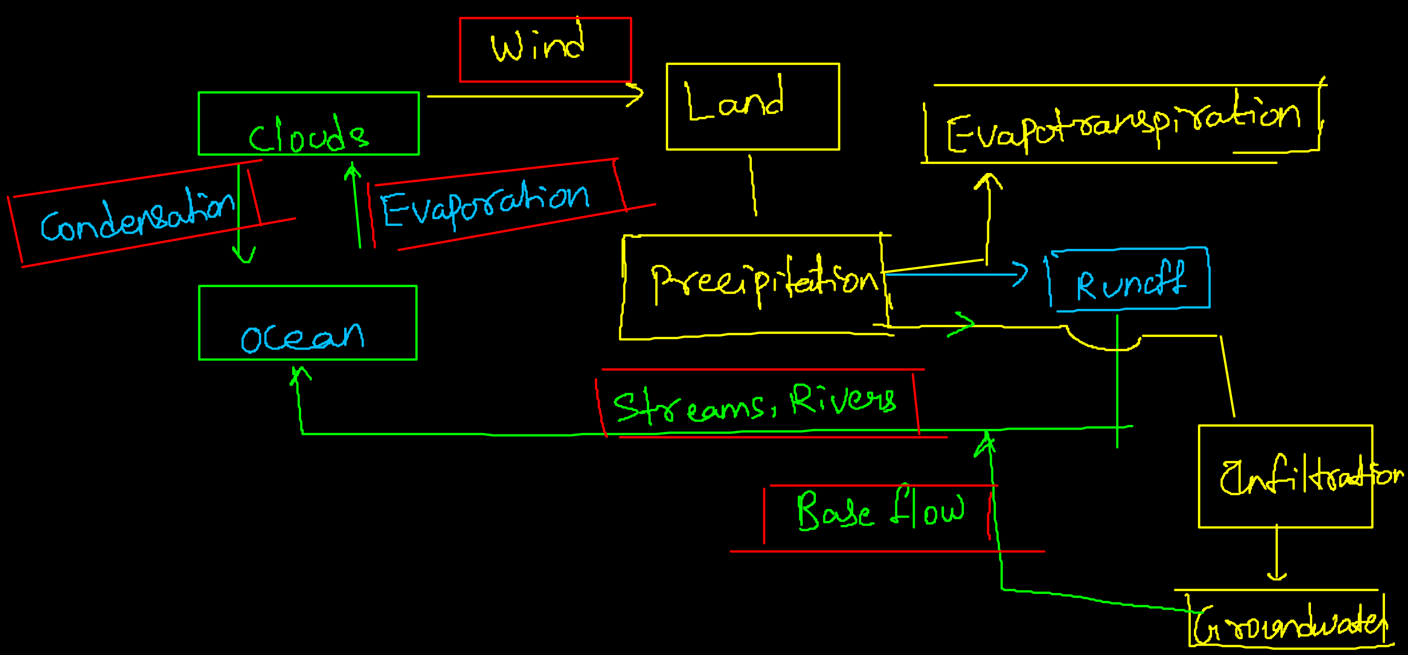
⇒ ⊛ study of occurence, distribution and circulation of the water

⇒ Hydrosphere ⇒

Lithosphere



⊛ Hydrologic cycle -- A closed loop which circulates the water across different phases.



★ Water Budget : Total amount of water on the Earth is constant. Continuity Eq^n can be applied.

$$\boxed{I - O = \Delta S}$$

I = Inflow Vol. in some catchment during Δt

O = outflow volume for some catchment during Δt

Δs = change in storage of the catchment during Δt

★ In a catchment

$$I - O = \Delta S$$

$$\Rightarrow \boxed{P - R - G - E - T = \Delta S} \Leftarrow \text{catchment}$$

For a lake

$$\boxed{P + I - Q - E - G = \Delta S}$$

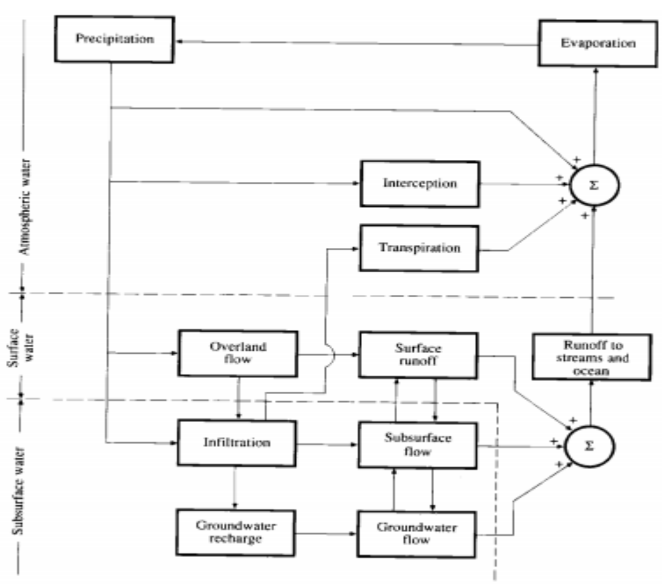


FIGURE 1.2.1
Block-diagram representation of the global hydrologic system.

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Ref: Chow et al. (1988)

Hydrologic cycle

Residence Time:- Water spends different amount of time in each part / phase of the cycle.

* It is the average time taken by the water to pass through a phase of the hydrologic cycle.
subsystem

$$T_r = \frac{\text{Vol. of water in that phase}}{\text{Net rate of flux.}}$$

* Global Rivers, $\boxed{T_r = 0.047 \text{ years} \approx 17.31 \text{ days}}$

* On an avg. water spends $\approx$ 17 days in a river.

* For atmospheric water, $\boxed{T_r \simeq 3 \text{ days}}$

↳ So, the atmospheric water is highly dynamic; So it is much difficult to model.

* For groundwater, $\boxed{T_r \approx 2 \text{ to } 3 \text{ years}}$

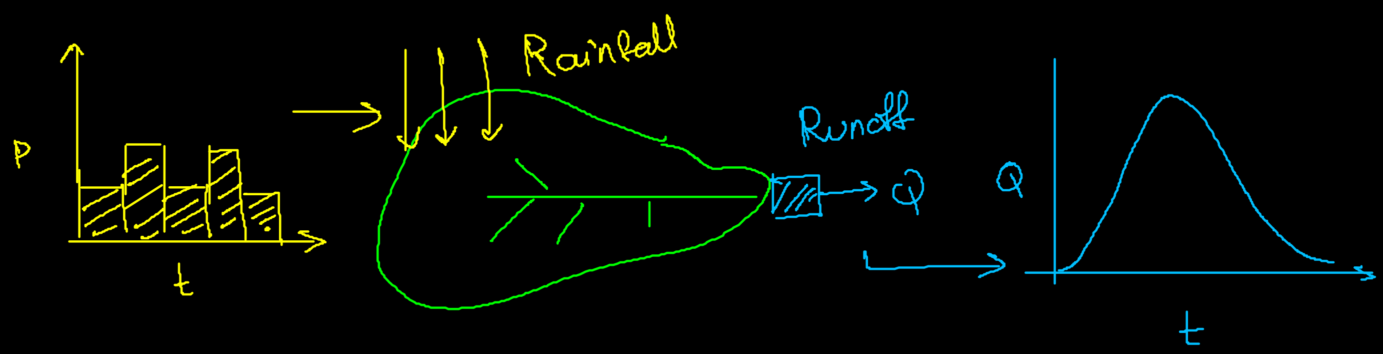
X XX X XX

Systems Concept in Hydrology

* A system should have → Input, output → $\boxed{\text{operator}}$

$$\boxed{\text{Input}} \longrightarrow \boxed{\text{operator}} \longrightarrow \boxed{\text{output}}$$

* A hydrolgic system is one in which the input and output are both hydrological variables.



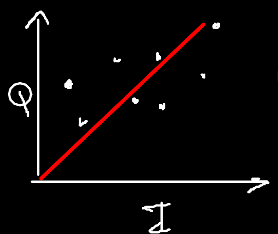
$$\boxed{Q(t) = \Omega\, I(t)}$$

$Q(t)$ = output
$I(t)$ = Input

Ω = Transfer function

* Constant Transfer function:-

$$Q(t) = C\, I(t)$$



$$C \Rightarrow \boxed{\Omega = \frac{Q(t)}{I(t)}}$$

Ex:- Rainfall, Runoff
Emperical methods.

Differential Transfer function

* Linear Reservoir is in which we assume that the storage is a linear function of the outflow,

$$\boxed{S = KQ}$$

* So, consider a linear reservoir.

→ We know that $\Delta S = I(t) - Q(t)$

$$\Rightarrow \frac{dS}{dt} = I(t) - Q(t).$$

$$\Rightarrow \frac{d(KQ)}{dt} = I(t) - Q(t)$$

$$\Rightarrow K \cdot \frac{dQ}{dt} = I(t) - Q(t)$$

$$\Rightarrow K \cdot \frac{dQ}{dt} + Q(t) = I(t).$$

Consider the operator $D = \frac{d}{dt}(\ )$

$$\Rightarrow (K \cdot D + 1) Q(t) = I(t)$$

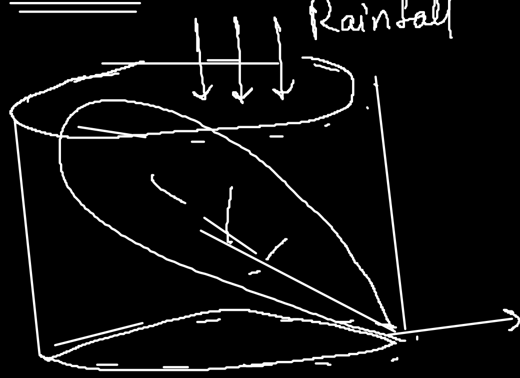
$$\Rightarrow Q(t) = \frac{1}{(KD+1)} I(t)$$

$$\Rightarrow \boxed{\Omega = \frac{1}{KD+1}}$$

⇒ For a linear reservoir.

Classification of Hydrological models.

- → Physical model → We have a full scale or scale model in the lab on which we simulate these processes.
  - → Scale Model
  - → Analog Model. (We use analogous process and build a model).
- → Abstract model.
  - → Here we use a set of equations of linking input and output variables.



$$I = f(x, y, z, t)$$

$$Q = f(x, y, z, t)$$

→ The Hydrological variables can be functions of space, time and an element of randomness.

$$I \longrightarrow \boxed{\begin{array}{c}\text{System}\\(x, y, z, t)\end{array}} \longrightarrow \text{output.}$$

$f(x, y, z, t, \varepsilon)$

Input → System → output

Random component

$y = mx + c + \varepsilon$

Deterministic — $y = mx + c$.

→ Same input, Same output

Stochastic

Same input can give diff. output

Deterministic → Lumped, Distributed

spatial variation ignored → Lumped; consider → Distributed

Lumped → steady, unsteady

Distributed → Steady, Unsteady

Stochastic → Space Independent, Space Correlated

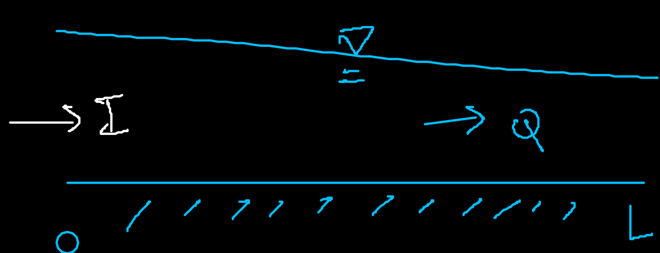
Space Independent: → Variables do not depend b/w any 2 sites

Space Correlated: → correlations exist in space.

Time Dep, Time. Corr. — $\times 2$

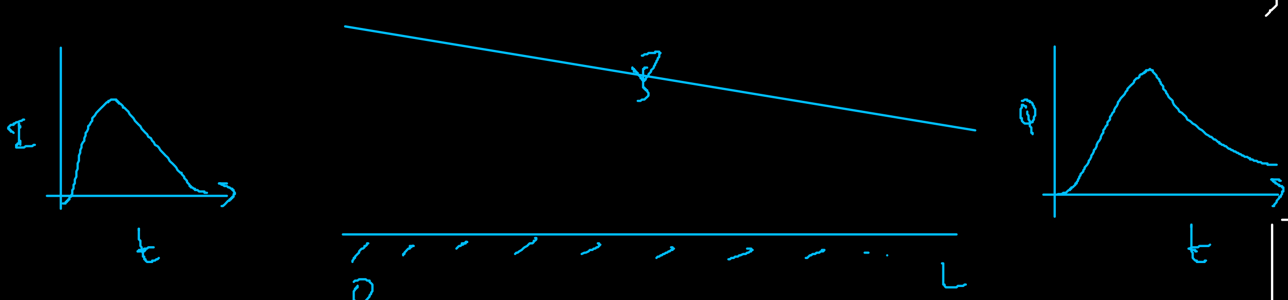
8 types of Models

Ex:-



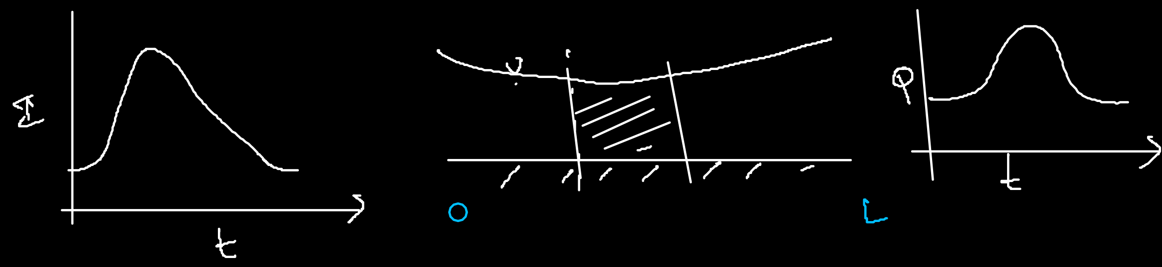
→ Deterministic Lumped and steady flow

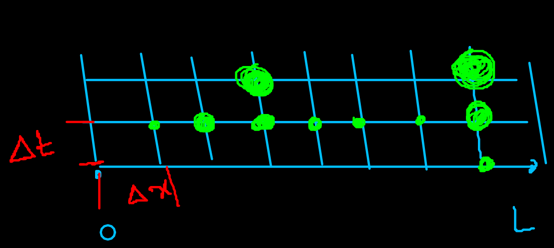
$$\boxed{I = Q}$$



→ Det. Lumped Unsteady

$$\boxed{\frac{ds}{dt} = I - Q}$$





⇓ Deterministic Distributed Unsteady Model

× ×× ——— ×××

HYDROLOGICAL PROCESSES

- Processes that transform the space and time distrb^n of water throughout the hydrologic scale.
  - Precipitation
  - Infiltration
  - Runoff
  - Evaporation etc
- We have 3 basic Eqns ⟶ Continuity, Momentum, Energy Eq^n which can model these processes.
- The basic theorem governing all these processes is called

$$\boxed{\text{Reynolds Transport Theorem}}$$

(R.T.T)

* Also called General control volume Theorem.

* Fluid properties → Extensive property (B)
  → Intensive property (β)

* Extensive property depends on mass of the fluid, intensive property is independent of the mass.

$\boxed{B = f(\text{mass})}$; Ex:- Momentum, Density etc.

$\boxed{\beta \neq f(\text{mass})}$; Ex:- Velocity, shape of fluid

and $\boxed{\beta = \frac{dB}{dm}}$ * β is the corresponding intensive property of the B.

⇒ $\boxed{\text{Intensive property} = \text{Extensive property} / \text{unit mass of fluid}}$

* These (B and β) can be either Scalar (or) Vector

* Lagrangian v/s Eulerian view

* If a camera is following the ball, this is the lagrangian view. (we move along with the particle)

* Eularian:- The camera is fixed, we look at certain frame and we analyze what happens to any particle crossing that reference.

> * Focus on particle : Lagrangian
>
> * Focus on fixed frame : Eularian in the fluid

* For RTT, we use Eularian approach.

↳ It relates $\boxed{\frac{dB}{dt}}$ to the external causes producing this change.

Time rate of change of extensive property.

$$\left(\frac{dB}{dt}\right) = \boxed{\text{Component 1}} + \boxed{\text{Component 2}}$$

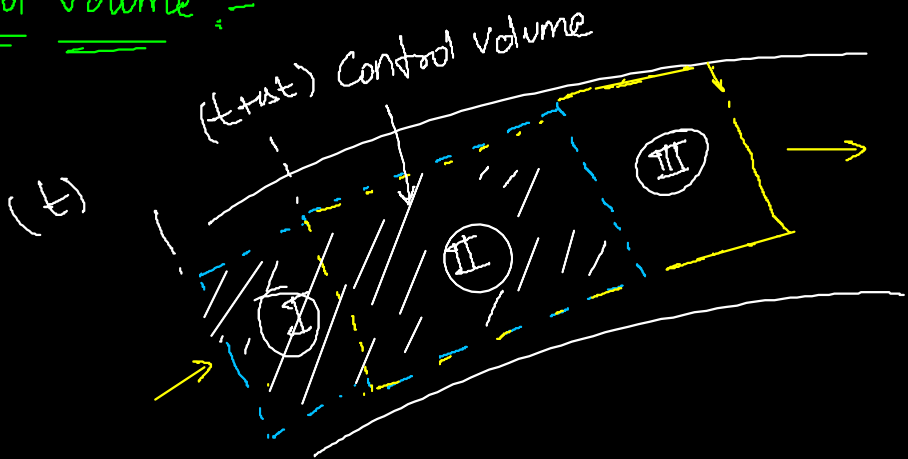
Component 1 ⇓ Time rate of change of 'B' stored within the control volume

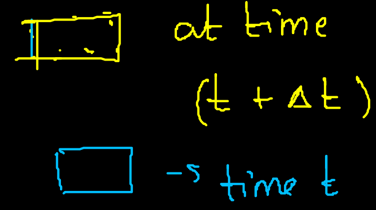
Component 2 ⇓ The net outflux of 'B' across the control surface

* RTT is used widely in Hydraulics and Fluid Mechanics, but it's use is limited in Hydrology.

Derivation

Control volume :-





(A) I = Region the fluid occupies at time t,

II = Region the fluid occupies at time t and $(t+\Delta t)$

III = Region the fluid occupies at time $(t+\Delta t)$

→ Consider elemental volume $= d\forall$; Density $= \rho$

$$\boxed{dm = \text{Mass} = \rho \, d\forall}$$

(A) Now, the amount of extensive property stored in the small volume $d\forall$

$$\Rightarrow \boxed{dB = \beta \cdot dm} \qquad \left(\text{Since } \beta = \frac{dB}{dm} \right)$$

⇒ Now, for the whole control volume.

$$\boxed{\text{Total amount of } B \text{ in the C.V} = \iiint_{C.V} \beta \rho \, d\forall} \quad \text{—— (1)}$$

(*) Now, time rate of change of extensive property is

$$\frac{dB}{dt} =$$

(*) $$\lim_{\Delta t \to 0} \left\{ \frac{1}{\Delta t} \left[\left(B_{II} + B_{III} \right)_{t+\Delta t} - \left(B_{I} + B_{II} \right)_{t} \right] \right\} \quad \text{—— (2)}$$

$\Rightarrow$ Rearranging.

$$\Rightarrow \frac{dB}{dt} = \lim_{\Delta t \to 0} \left\{ \underbrace{\frac{1}{\Delta t} \left[\left(B_{II} \right)_{t+\Delta t} - \left(B_{II} \right)_{t} \right]} + \frac{1}{\Delta t} \left[\left(B_{III} \right)_{t+\Delta t} - \left(B_{I} \right)_{t} \right] \right\} \quad \text{—— (3)}$$

As $\Delta t \to 0$, Region (II) will coincide with C.V

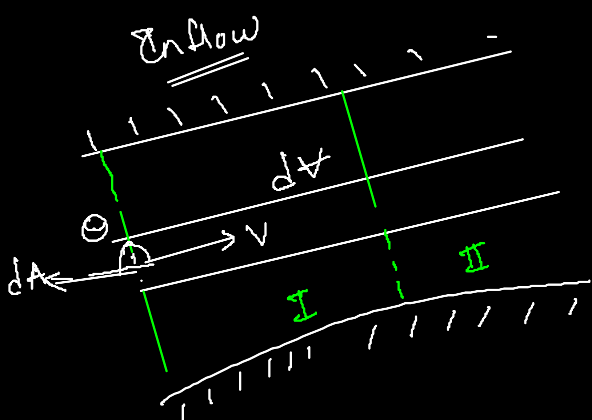
$\Downarrow$

$$\lim_{\Delta t \to 0} \left\{ \frac{1}{\Delta t} \left[\left(B_{II} \right)_{t+\Delta t} - \left(B_{II} \right)_{t} \right] \right. = \boxed{\frac{d}{dt} \iiint_{C.V} \beta \rho \, d\forall}$$

Now the 2nd term in Eqn (3)

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \left(\left(B_{III} \right)_{t+\Delta t} - \left(B_{I} \right)_{t} \right) = ?$$

Ⓐ Expandend view of outflow region

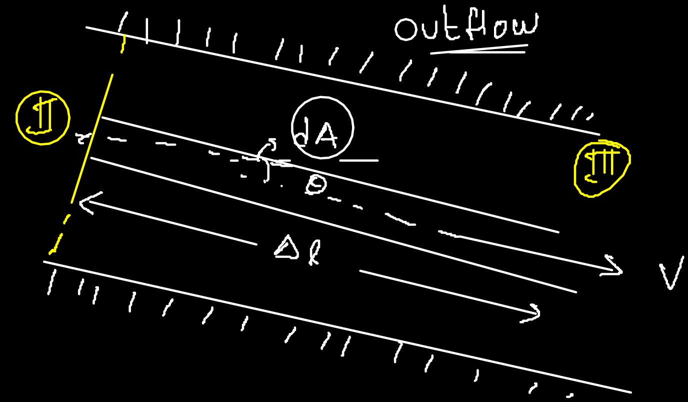


$$\vec{V}\cdot d\vec{A} = V\cos(180-\theta)\times dA$$

Similarly at the inlet, we get

$$\lim_{\Delta t\to 0} \frac{(B_{I})_{\Delta t}}{\Delta t} = \iint_{c\cdot s} \beta\rho\, \vec{V}\cdot d\vec{A}$$

Ⓐ Substitute all expressions



Ⓐ Elemental volume

$$d\forall = dA\cdot\Delta l\cdot\cos\theta$$

Δl = Velocity $\times$ Δt

$$\vec{V}\cdot d\vec{A} = V\cos\theta\cdot dA$$

Total amount of B

$$= \iint_{c\cdot s} \beta\rho\,(dA\cdot\Delta l\cdot\cos\theta)$$

$$\lim_{\Delta t\to 0} \frac{(B_{III})_{t+\Delta t}}{\Delta t} = \frac{d}{dt}\iint \beta\rho\, dA\, \Delta l\cos\theta$$

But $\lim_{\Delta t\to 0} \frac{\Delta l}{\Delta t} = \vec{V}$

$$\Rightarrow \boxed{\iint_{III} \beta\rho\, \vec{V}\cdot d\vec{A}}$$

$$\frac{dB}{dt} = \frac{d}{dt}\iiint_{C\cdot V} \beta\rho\, d\forall + \underbrace{\iint_{\mathrm{II}} \beta\rho \vec{V}\cdot d\vec{A} - \iint_{\mathrm{I}} \beta\rho\vec{V}\cdot d\vec{A}}_{\text{Flux across the C.S}}$$

$$\Rightarrow \boxed{\underbrace{\frac{dB}{dt}}_{\substack{\text{Time rate of} \\ \text{change of B}}} = \underbrace{\frac{d}{dt}\iiint_{C\cdot V} \beta\rho\, d\forall}_{\substack{\text{Time rate of change} \\ \text{of B stored in C.V}}} + \underbrace{\iint_{C\cdot S} \beta\rho\vec{V}\cdot d\vec{A}}_{\substack{\text{Net flow of B} \\ \text{across the C.S}}}}$$

- For impermeable condition $\vec{V}\cdot d\vec{A} = 0$; b/c $\theta = 90^\circ$
- Inflow : $90^\circ < \theta < 270^\circ \Rightarrow \cos\theta = -ve$
  (Inflow treated $-ve$)
- outflow, $\theta < 90^\circ \Rightarrow \cos\theta = +ve$
  ↳ (outflow treated $+ve$)

- Now, we can apply the RTT to derive the continuity, momentum and energy equations.

⊛ Continuity Equation :- Integral form.

→ It is the representation of law of conservation of mass.

→ So, we consider mass as the extensive property.

$$\frac{dB}{dt} = \frac{d}{dt}\iiint_{C\cdot V} \beta \rho \, d\forall + \iint_{C\cdot S} \beta \rho \, \vec{V} \cdot \vec{dA}$$

Here $\beta = \frac{dB}{dm} = \frac{d(m)}{dm} = \underline{\underline{1}}$.

$$\Rightarrow \frac{dB}{dt} = \frac{d}{dt}\iiint_{C\cdot V} \rho \, d\forall + \iint_{C\cdot S} \rho \vec{V} \cdot \vec{dA}$$

also, $\boxed{\frac{dm}{dt} = 0} \Rightarrow \boxed{\frac{dB}{dt} = 0}$

Law of Mass Conservation →

$$\Rightarrow \boxed{0 = \frac{d}{dt}\iiint_{C\cdot V} \rho \, d\forall + \iint_{C\cdot S} \rho \vec{V} \cdot \vec{dA}}$$

⋆ For unsteady

⋆ compressible.

⊛ If ρ = Constant, ⇒ Incompressible

$$\Rightarrow 0 = \boxed{\frac{d}{dt}\iiint d\forall} + \boxed{\iint_{C\cdot S} \vec{V} \cdot \vec{dA}} \quad \text{—— (10)}$$

$$\Rightarrow \iiint d\forall = \text{Storage} \Rightarrow \frac{d}{dt}\iiint d\forall = \boxed{\frac{ds}{dt}}$$

$$\iint_{C.S} \vec{V}\cdot d\vec{A} = \text{Volumetric Flow across the surface}$$

$$= \iint_{\text{outlet}} \vec{V}\cdot d\vec{A} - \iint_{\text{inlet}} \vec{V}\cdot d\vec{A}$$

$$= Q(t) - I(t).$$

$$\Rightarrow \boxed{\frac{ds}{dt} = I(t) - Q(t)}$$

* Incompressible
* Unsteady

* If we consider, Steady state $\Rightarrow ds/dt = 0$

$$\Rightarrow \boxed{Q(t) = I(t)}$$

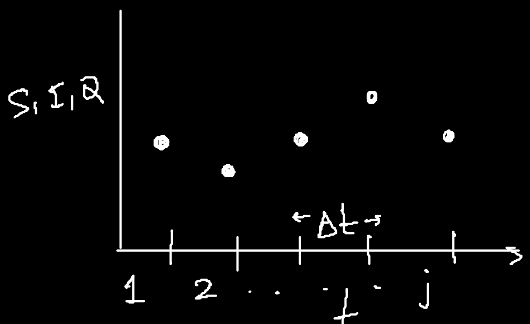
* This applies to only a single phase flow. For a multiphase flow, we need to write continuity Eq^n for each phase.

In a closed system total amounts of inflow and outflow are equal
Example: Hydrologic cycle

$$\int_{-\infty}^{\infty} I(t) = \int_{-\infty}^{\infty} Q(t)$$

In a open system total amounts of inflow and outflow are not equal
Example: Rainfall-runoff process on a watershed

Discrete time continuity Eqn:- When we measure, the data we have are at discrete time steps. We write the continuity Eqn at discrete time steps.



$$\frac{dS}{dt} = I(t) - Q(t)$$

$$\Rightarrow dS = I(t)\,dt - Q(t)\,dt$$

$$\Rightarrow \int_{S_{j-1}}^{S_j} dS = \int_{(j-1)\Delta t}^{(j\Delta t)} I(t)\,dt - \int_{(j-1)\Delta t}^{j\Delta t} Q(t)\,dt$$

$$\Rightarrow \boxed{(S_j - S_{j-1}) = I_j - Q_j}$$

I_j = volume of inflow during time interval Δt

Q_j = " " outflow " " " Δt.

$$\boxed{\Delta S_j = I_j - Q_j}$$

$$\Rightarrow \boxed{S_j = S_{j-1} + (I_j - Q_j)}$$

for $j = 1, 2, 3 \ldots n$

At $j = 1 \Rightarrow S_1 = S_0 + (I_1 - Q_1)$

S_0 → Initial storage.

At $j=2$

$$S_2 = S_1 + (I_2 - Q_2)$$

$$\Rightarrow S_2 = S_0 + (I_1 - Q_1) + (I_2 - Q_2)$$

$$\vdots$$

$$\boxed{S_n = S_0 + (I_1 + I_2 + \dots I_n) - (Q_1 + Q_2 + \dots Q_n)}$$

$$\Rightarrow \boxed{S_n = S_0 + \sum_{j=1}^{n} (I_j - Q_j)}$$ $\Rightarrow$ Discrete time cont. Eq^n

⊛ Momentum Equation

→ Here the extensive property = Momentum = $(B = m\vec{V})$

$$\Rightarrow \beta = \frac{dB}{dm} = \vec{V}$$

and $$\boxed{\frac{dB}{dt} = \frac{d}{dt}\iiint_{C.V} \beta\rho\, d\forall + \iint_{C.S} \beta\rho\,\vec{V}\cdot d\vec{A}}$$

$$\frac{dB}{dt} = \boxed{\frac{d}{dt}(m\vec{V}) = \Sigma F}$$ (From Newton's Law) 2nd

$$\Rightarrow \boxed{\sum F = \frac{d}{dt}\iiint_{C.V} \vec{V}\cdot \rho\, d\forall + \iint_{C.S} \vec{V}\rho\,\vec{V}\cdot d\vec{A}}$$

* Unsteady
* Non-uniform
* Compressible

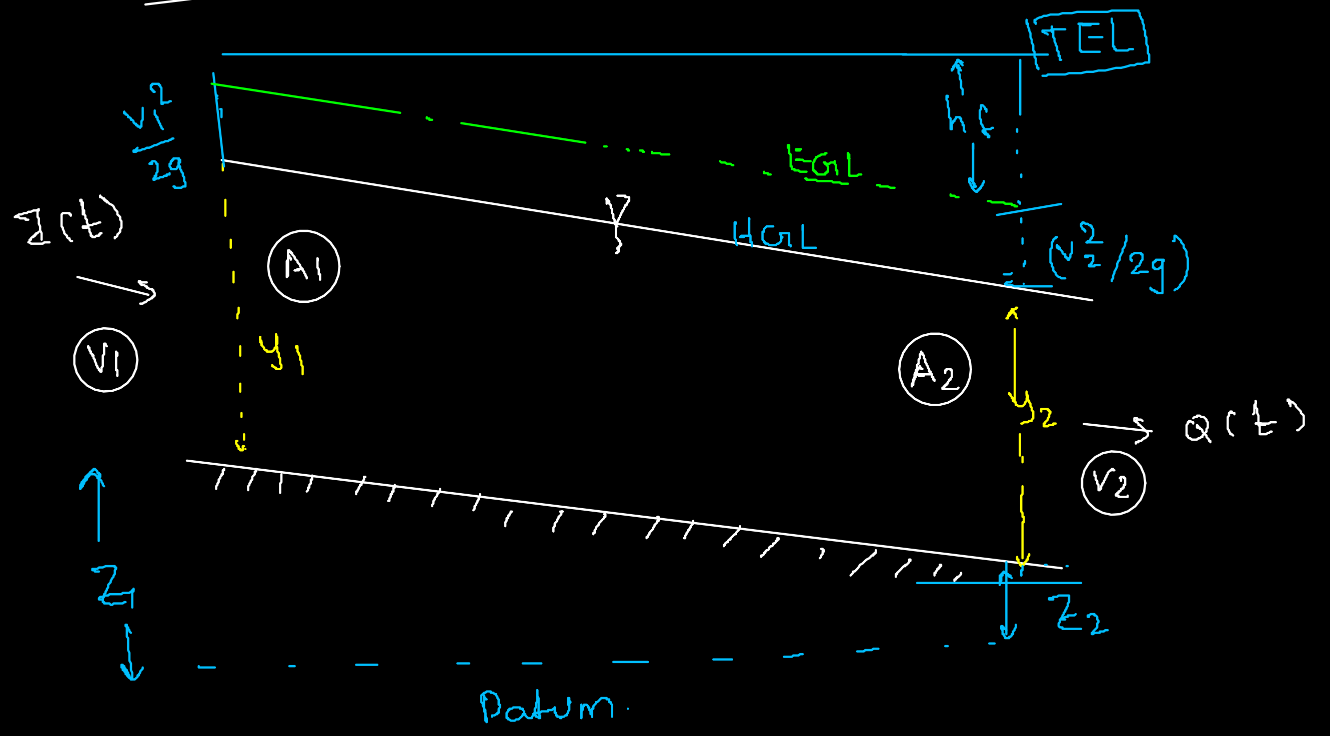
* For steady flow, incompressible

$$\Rightarrow \boxed{\sum F = \iint_{C.S} V \rho\, \vec{V}\cdot d\vec{A}}$$

* Steady and uniform, $\vec{V}\cdot d\vec{A} = 0$ (Velocity not changing) incompressible

$$\boxed{\sum F = 0}$$

1-D Steady Uniform flow in open channel



Continuity Eq^n

$$\frac{ds}{dt} = I(t) - Q(t)$$

For steady case

$ds/dt = 0$

$$\Rightarrow \quad I(t) = Q(t)$$

$$\Rightarrow \quad \boxed{Q_1 = Q_2}$$

Uniform flow

$V_1 = V_2$

$$\Rightarrow \quad \boxed{A_1 = \frac{Q}{V_1}} \text{ and } \boxed{A_2 = \frac{Q}{V_2}}$$

$\Rightarrow \boxed{A_1 = A_2}$ because $V_1 = V_2$ $\Rightarrow \boxed{y_1 = y_2}$ → For uniform flow

Energy Equation

$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g} + h_f$$

Head Loss

Since, $y_1 = y_2$ and $V_1 = V_2$

$$\Rightarrow \quad Z_1 = h_f + Z_2$$

$$\Rightarrow \quad \boxed{h_f = Z_1 - Z_2}$$

Bed slope

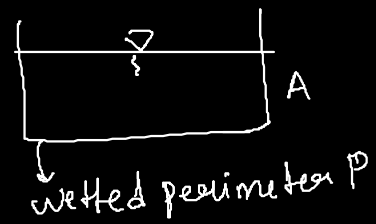
$$\Rightarrow \quad \frac{h_f}{L} = \frac{Z_1 - Z_2}{L} \Rightarrow \boxed{S_0 = \frac{h_f}{L} = S_f}$$

(*) For small θ, we assume $\tan\theta = \sin\theta \approx \theta \Rightarrow \boxed{S_o = S_f}$

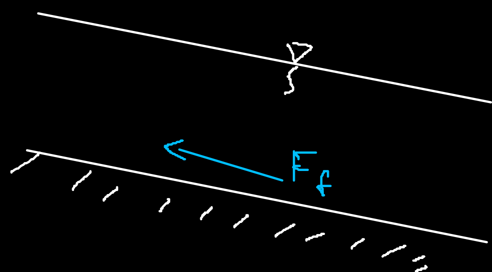
(A) **Momentum Eqⁿ** $\quad \sum F = 0$

$$\Rightarrow F_g + F_p + F_f = 0 \quad \text{(Gravity, pressure, frictional)}$$

→ The $\boxed{F_p = 0}$ in this case as $y_1 = y_2$



(*) **Frictional force**



* Bed and side of the channel cause friction

$$F_f = -\tau_o \, P \cdot L$$

τ_o → wall shear stress.

$P \cdot L$ → Area = wetted perimeter × Length.

Gravity forces:-

F_g = Component of water weight in x-dirⁿ

$$F_g = (\gamma \, A L) \sin\theta$$

γ → wt. Density. $\quad AL$ → volume

So, now $\sum F = 0$

$$\Rightarrow \gamma A L \sin\theta + 0 - P L \tau_o = 0$$

$$\Rightarrow \tau_0 = \frac{\gamma A L \sin\theta}{PL} = \frac{\gamma A \sin\theta}{P}$$

since $(A/P) = R$

$$\Rightarrow \boxed{\tau_0 = \gamma R S_0}$$

When θ is small

$\boxed{S_0 \approx S_f.}$

(*) __Darcy Law__ :- In groundwater system the flow through porous media is equivalent to pipe flow of diameter 'd'.

So, we assume $\boxed{\tau_0 = \gamma R S_0}$; Pipe flow $\Rightarrow \boxed{R = D/4}$

(*) For Laminar flow in circular pipe

$$\boxed{\tau_0 = \frac{8\mu V}{D}}$$

μ = Dynamic Viscosity

V = Velocity.

$$\Rightarrow \frac{8\mu V}{D} = \gamma R S_0$$

$(S_0 \approx S_f)$

$$\Rightarrow \frac{8\mu V}{D} = \gamma \cdot \frac{D}{4} \cdot S_0 \Rightarrow \boxed{V = \left(\frac{\gamma D^2}{32\mu}\right) S_f}$$

K

$\Rightarrow \boxed{V = KS_f}$; But $V = Q/A \approx q$

$$\Rightarrow \frac{Q}{A} = KS_f$$

$\Rightarrow \boxed{q = KS_f}$ or $\boxed{q = Ki}$

* But the actual flow velocity.

$$\boxed{V_a = \frac{q}{\eta}}$$

η = Porosity.

$\boxed{V_a > V}$

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* Energy Equation

In RTT, we consider $\boxed{B}$ = Energy = $\Big(\overset{E}{\overline{\text{Internal energy}}} + \underbrace{K.E}_{\frac{1}{2}mv^2} + \underbrace{P.E}_{mgZ}\Big)$

$$\Rightarrow \beta = \left(E_U + \frac{V^2}{2} + gZ\right)$$

* E_U = Internal energy/ unit mass.

$\Rightarrow$ 1st law of Thermodynamics.

$$\boxed{\frac{dB}{dt} = \frac{dH}{dt} - \frac{dW}{dt}}$$

H = Heat transferred to water

W = Work done by the fluid on the system.

⊛ Now, RTT

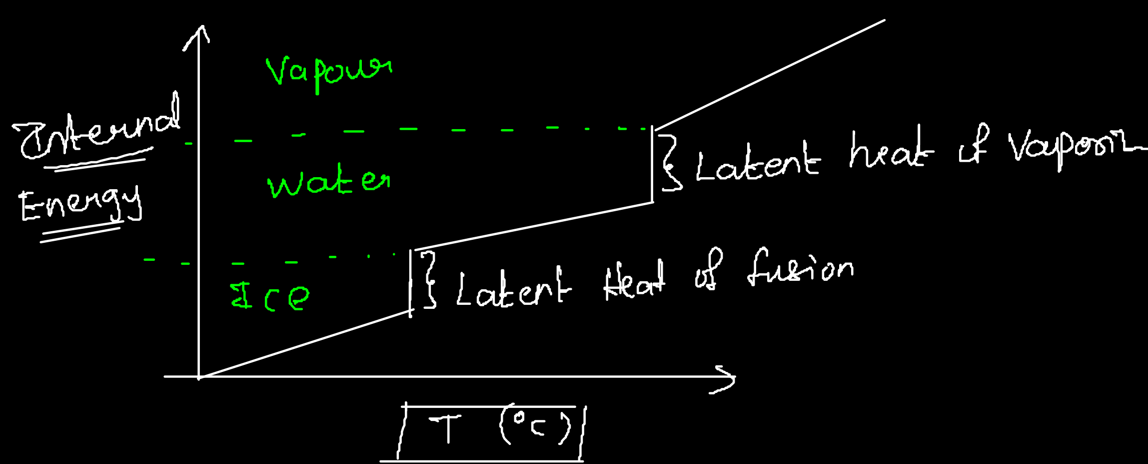
$$\frac{dH}{dt} - \frac{dW}{dt} = \frac{d}{dt}\iiint_{C.V}\left(E_U + \frac{V^2}{2} + gz\right)\rho\, d\forall + \iint_{C.S}\left(E_U + \frac{V^2}{2} + gz\right)\rho\, \vec{V}\cdot\vec{dA}$$

⇒

⊛ Internal Energy → Sensible Heat
→ Latent Heat

⊛ Sensible Heat is that component which depends on temperature

⊛ Latent Heat can be used by a fluid for phase change
↳ Vaporization, Sublimation



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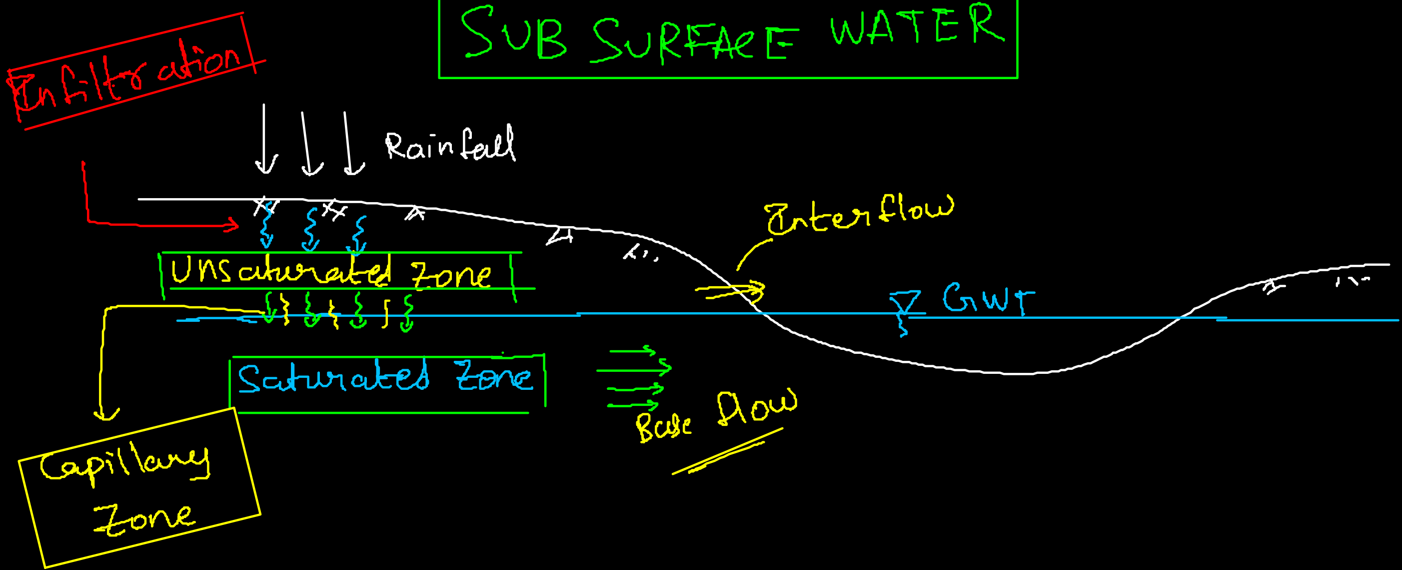
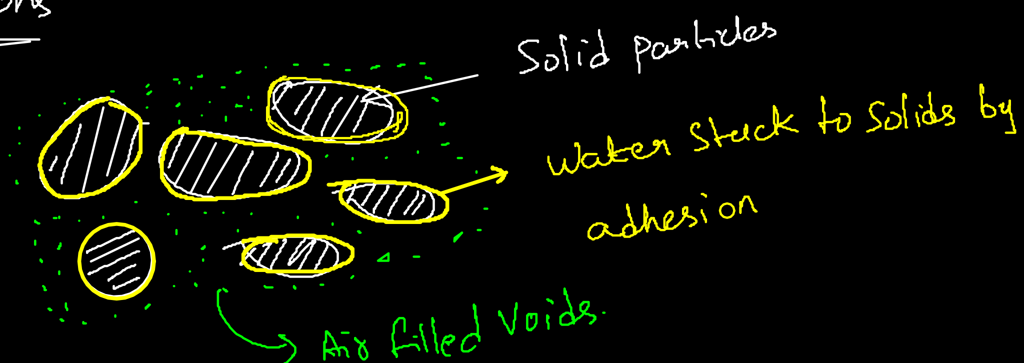
③ ways of Heat Transfer → Conduction
→ Convection
→ Radiation

SUB SURFACE WATER



(A) Basic Definitions



(B) There are 3 dominant forces: Gravity, Friction, Suction.

(A) Below GWL → only gravity and friction

(A) Porosity (n) = Ratio of Volume of voids to total volume of the Soil Sample.

$$\boxed{n = \frac{\text{Vol. of Voids}}{\text{Total Vol.}}}$$

$\boxed{n \text{ varies b/w } 0 \text{ and } 1}$

⇒ Porosity and θ → $\boxed{\text{Dimensionless}}$

⊛ $\boxed{\text{Moisture content } (\theta) = \dfrac{\text{Vol. of water}}{\text{Total volume}}}$

'θ' can vary b/w $\boxed{0 \text{ and } n}$

⊛ $\boxed{\text{For a saturated Soil, } \theta = n;\ \text{dry soil; } \theta = 0}$

1-D unsteady unsaturated flow (Richard's Eq^n)

↳ continuity Eq^n + Momentum Eq^n

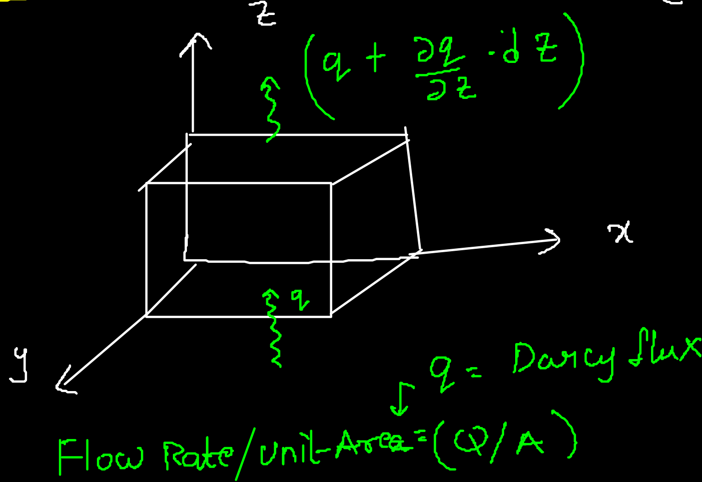
⊛ Continuity Eq^n:-

Z (+ve) upwards

Vol = $(dx\,dy\,dz)$

Vol. of water

$= \theta(t)\,dx\,dy\,dz$



Consider an elemental volume with sides dx, dy, dz. the moisture content $= \theta$

⊛ Here, to apply RTT, the extensive property, B = Mass of Soil water.

and $\beta = dB/dm = 1$

⊛ From law of conservation of mass ⇒

$$\boxed{\frac{dB}{dt} = 0}$$

$$\Rightarrow \quad 0 = \frac{d}{dt}\underbrace{\iiint_{C \cdot V} \rho_w \, d\forall}_{①} + \underbrace{\iint_{C \cdot S} \rho_w \vec{V} \cdot d\vec{A}}_{②}$$

ρ_w = Density of water

① $\Rightarrow$

$$\frac{d}{dt} \iiint_{C \cdot V} \rho_w \, d\forall = \frac{d}{dt} \iiint_{C \cdot V} (\rho_w \, dx \, dy \, dz)\,\theta$$

$$① = \underline{\underline{\rho_w}}\, \underline{\underline{dx\, dy\, dz}} \cdot \underline{\underline{\left(\frac{\partial \theta}{\partial t}\right)}}$$

② $\Rightarrow$

$$\iint_{C \cdot S} \rho_w \vec{V} \cdot d\vec{A}$$

$\rightarrow$ we have inflow $= q$, Area $= (dx\,dy)$

outflow $= q + \frac{\partial q}{\partial z} \cdot dz$, Area $= (dx\,dy)$

$$\Rightarrow \iint_{C \cdot S} \rho_w \left(q + \frac{\partial q}{\partial z} \cdot dz\right) dx\, dy - \rho_w q \, dx\, dy$$

$$② \Rightarrow \underline{\underline{\rho_w}}\, \underline{\underline{dx\, dy\, dz}} \underline{\underline{\left(\frac{\partial q}{\partial z}\right)}}$$

① + ② = 0

$$\Rightarrow \boxed{\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0}$$

* The time rate of change of soil moisture content = vertical gradient of Darcy flux.

1D-Unsteady unsaturated flow continuity Eqn

$$\boxed{\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z}}$$

* Momentum Eqn : We know that $q = KS_f$

$\boxed{q = KS_f}$; Here q = Darcy's Flux.

S_f = Rate of head loss/unit length.

⇒ The flow is taking place in vertical direction

$$q = \left(-K \cdot \frac{\partial h}{\partial z}\right)$$

* Now, the head (h) consists of 3 components
  - → Suction head
  - → Gravity head
  - → Velocity head

$$h = \left(\psi + z + \frac{v^2}{2g}\right)$$

ψ = Suction head

z = Datum head

$\frac{v^2}{2g}$ = Velocity head

* In subsurface $\frac{v^2}{2g} \approx 0$ (very small)

$$\Rightarrow h = (\psi + z)$$

$$\boxed{q = -K \cdot \frac{\partial(\psi + z)}{\partial z}}$$

$$\frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial \theta} \cdot \frac{\partial \theta}{\partial z}$$

using chain rule.

$$\Rightarrow q = \left(-K \cdot \frac{\partial \psi}{\partial \theta} \cdot \frac{\partial \theta}{\partial z} + K\right)$$

$$\boxed{D = K \cdot \frac{\partial \psi}{\partial \theta}} \Leftarrow \text{Soil water Diffusivity } (L^2/T)$$

$$\Rightarrow q = \left(-D \frac{\partial \theta}{\partial z} + K\right)$$

(A) substituting this in the continuity Eqn

$$\Rightarrow \frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z}\left(-D \frac{\partial \theta}{\partial z} + K\right)$$

$$\Rightarrow \boxed{\frac{\partial \theta}{\partial t} = \underbrace{D \frac{\partial^2 \theta}{\partial z^2}}_{\text{Diffusion of soil moisture}} + \underbrace{D \frac{\partial K}{\partial z}}_{\text{Gradient of permeability}}}$$

Richards Eqn

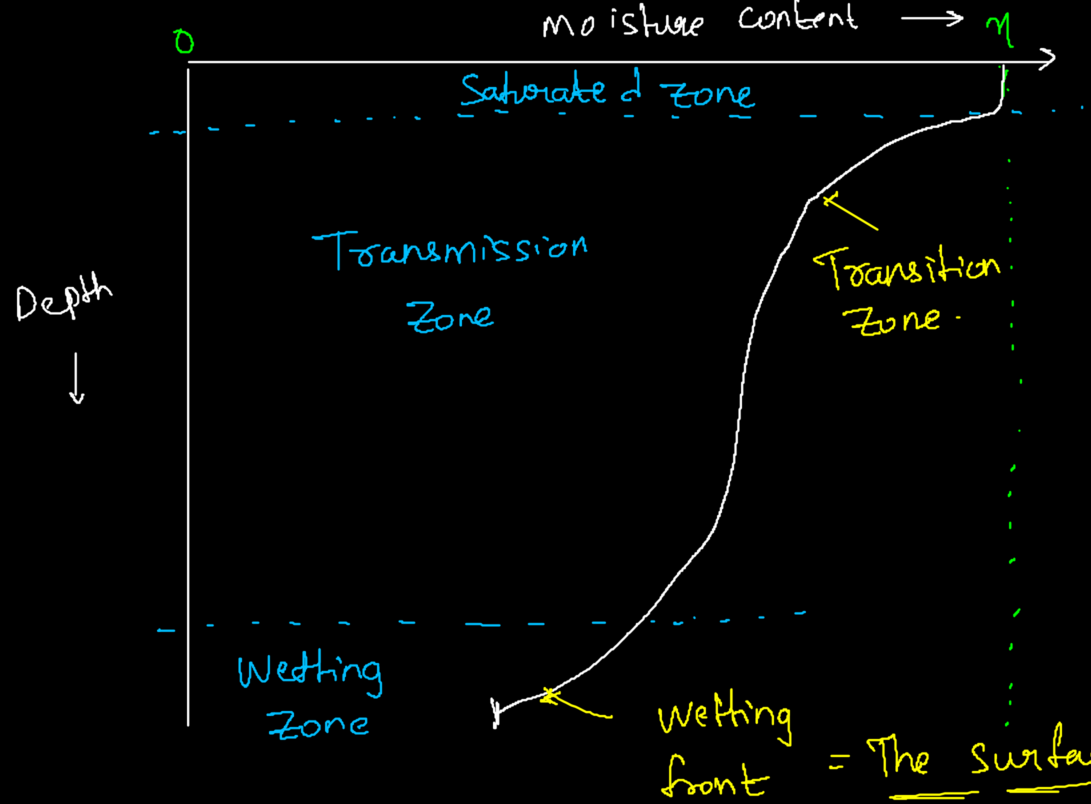
$\Rightarrow$. Rate of change of Soil moisture

Infiltration

* Depends on Soil properties and the surface condition.

↳ Also depends on the Antecedent moisture content (AMC)

* Soil moisture Profile



= The surface where 'θ' decreases very sharply.

* The infiltration rate = $f(t)$ at any given time.

Total cumulative infiltration ⇒ $\boxed{\int_0^t f(\tau)\,d\tau = F(t)}$

⇒ $\boxed{f(t) = \dfrac{dF}{dt}}$

* Most infiltration equations give us the **potential infiltration** ⇓ Amount of infiltration when water is ponded on soil surface

↳ Then how to find actual infiltration.

* Actual infiltration Rate = f_a

  Potential Infiltration = f_p

  Rainfall intensity = i

⟹ If $i > f_p$ ⟹ Then $\boxed{f_a = f_p}$

$i < f_p$ ⟹ Then $\boxed{f_a = i}$

* **Horton's Infiltration Eq^n**

$$\boxed{f(t) = f_c + (f_0 - f_c)e^{-kt}}$$

* $f(t)$ = Rate of Infiltration at any time t

  f_c = steady state infiltration, f_0 = Initial Infiltration

  k = exponential decay constant.

* If we assume that $\boxed{\text{'K' and 'D' are independent}}$ $\boxed{\text{of '}\theta\text{'}}$, then → Soil properties

Horton's Eq^n can be derived from Richards Eq^n

$$\boxed{\frac{\partial \theta}{\partial t} = D \cdot \frac{\partial^2 \theta}{\partial z^2}}$$

⇐ Solved to get Horton's Eq^n

Philip's Equation

* Here, we assume that K and D can vary with θ. We use Boltzman transformation, $B(\theta) = Zt^{-1/2}$

$$\boxed{F(t) = \underbrace{St^{1/2}}_{\text{suction term}} + \underbrace{kt}_{\text{Gravity term}}}$$

S = Sorptivity is a f^n of soil suction

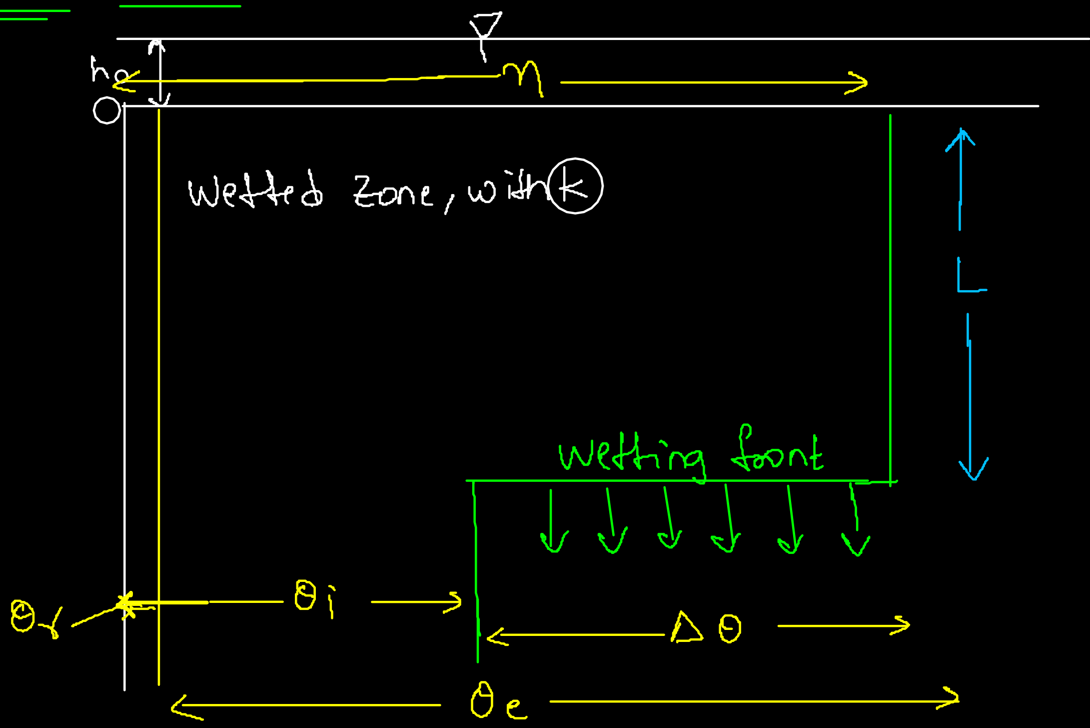
$$\Rightarrow \boxed{f(t) = \frac{1}{2} S t^{-1/2} + K}$$

As $t \to \infty$, $\boxed{f(t) \to k}$

* For a horizontal column of water, only suction forces are present $\Rightarrow \boxed{F(t) = St^{1/2}}$

Green-Ampt Method :- It provides the analytical solution to the Richard's Eqn with some assumptions.

Snapshot at any time



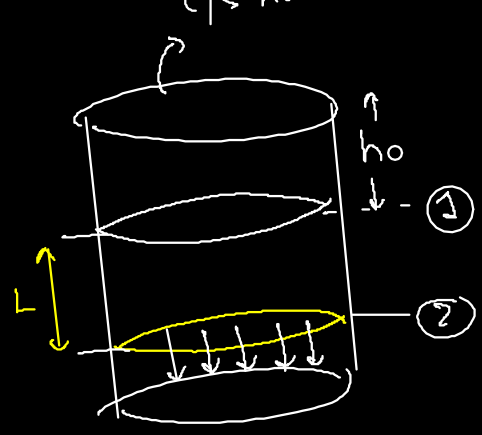
θ_i = Moisture content below the wetting front

η = Porosity, θ_r = Residual M.C; L = Depth of wetting front at time 't'

h_0 = Ponding depth.

$\boxed{\theta_e = (\eta - \theta_r)}$; effective porosity

① Continuity



C.V ≡ cylinder of length L.

Let θ_i = Initial M.C ; η = Porosity

θ = M.C at any time.

$\boxed{\theta_i < \theta < \eta}$

* Amount of water stored in the C.V due to infiltration at any time 't'

① Volume of CV = $(L \times 1)$

Amount of water = $(L \times 1) \times \Delta\theta$

$$\Rightarrow (L \times 1) \times (\eta - \theta_i)$$

$$\Rightarrow \boxed{L(\eta - \theta_i) = F(t)}$$

$$\Rightarrow \boxed{F(t) = L\Delta\theta} \qquad \boxed{\Delta\theta = \eta - \theta_i}$$

② Momentum Eqⁿ

$$q = -K\left(\frac{\partial h}{\partial z}\right)$$

Since q is +ve upwards, -ve ↓ ; f is +ve, downward.

$$\Rightarrow f = K\left(\frac{h_1 - h_2}{z_1 - z_2}\right)$$

$\boxed{h_2 = -\psi - L}$

Notation, -ve downward.

B/w eq's ① and ②.

$$f = K\left[\frac{(h_0) - (-\psi - L)}{L}\right]$$

$$\boxed{f = K\left(\frac{\psi + L}{L}\right)}$$

If $h_0 << L$

(A) we know that $F = L\Delta\theta \Rightarrow L = F/\Delta\theta$

$\hookrightarrow$ continuity

$$\Rightarrow \boxed{f = K\left[\frac{\psi\Delta\theta + F}{F}\right]}$$

But $f = dF/dt$

$$\Rightarrow \frac{dF}{dt} = \frac{K\psi\Delta\theta}{F} + \frac{KF}{F}$$

$$\Rightarrow \left(\frac{F}{F+\psi\Delta\theta}\right) dF = K \cdot dt$$

Add and subtract $\psi\Delta\theta$

$$\Rightarrow \left(\frac{F+\psi\Delta\theta - \psi\Delta\theta}{F+\psi\Delta\theta}\right) dF = K dt$$

$$\Rightarrow \int_0^{F(t)} \left(1 - \frac{\psi\Delta\theta}{F+\psi\Delta\theta}\right) dF = \int_0^{t} K dt$$

$$\Rightarrow F(t) - \psi\Delta\theta \ln\left(1 + \frac{F(t)}{\psi\Delta\theta}\right) = Kt$$

Green Ampt Eqn for cumulative Infiltration $\Rightarrow$

$$\boxed{F(t) - \psi\Delta\theta \ln\left(1 + \frac{F(t)}{\psi\Delta\theta}\right) = Kt}$$

$\hookrightarrow$ can be solved by iteration

Once we have $F(t)$;

$$\boxed{f(t) = K\left(\frac{\psi\Delta\theta}{F(t)} + 1\right)}$$

(*) If h_0 is not negligible, $(\psi = \psi - h_0)$

(*) Effective saturation (S_e)

$$S_e = \frac{\text{Available moisture}}{\text{Maximum possible available}}$$

$$\Rightarrow \boxed{S_e = \frac{\theta - \theta_r}{\eta - \theta_r}}$$

But $\eta - \theta_r = \theta_e$ effective porosity.

When $\theta_r \leq \theta \leq \eta \Rightarrow \boxed{0 \leq S_e \leq 1}$

$$\Rightarrow \boxed{\theta_i - \theta_r = S_e \cdot \theta_e}$$

(*)

Now, $\Rightarrow \Delta\theta = \eta - \theta_i$

$$\Rightarrow \Delta\theta = (\theta_r + \theta_e) - (\theta_r + S_e \cdot \theta_e)$$

$$\Rightarrow \boxed{\Delta\theta = \theta_e(1 - S_e)}$$

* The green ampt parameters are $\underbrace{K, \eta, \psi}$ and the initial conditions (S_e) should be known.

↳ we consider them as constants.

* However $\psi = f(\theta)$

Brooks and Corey showed that both ψ and $K = f(\theta)$.

Atmospheric water

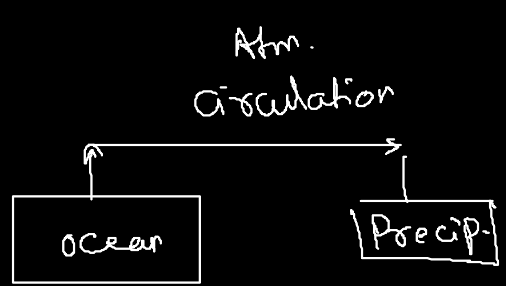
↳ Gaseous (vapor)

↳ Liquid water (Rain

↳ Solid (snow, hail etc)

Residence time of atmospheric water is very less $\approx$ 7 days.

* Atmospheric circulation



② factors

↳ Heat energy from sun

↳ Rotation of the Earth.

↳ Irregular heating b/w poles and equator.

↳ Causes winds

⊛ The amount of water in the atmosphere ≈ 0.001 % of total available water.

⊛ Movement of water in the air

B = Mass of water vapor.

⊛ The fluid = Moist Air = Water Vapour + Dry air.

$$\beta = \frac{dB}{dm} = \frac{\text{Mass of water vapor}}{(\text{Mass of water vapor} + \text{dry air})}$$

$$\Rightarrow \boxed{\beta = q_v} \Leftarrow \boxed{\text{Specific humidity}}$$

$$\Rightarrow q_v = \frac{\text{Mass of WV}}{\text{Mass of WV} + \text{dry air}} = \frac{\rho_v \cdot \text{Vol}}{\rho_a \cdot \text{Vol}}$$

$$\Rightarrow \boxed{q_v = \frac{\rho_v}{\rho_a}}$$

⊛ Ratio of density of water vapor to density of moist air

⊛ $$\frac{dB}{dt} = \frac{d}{dt}\iiint_{cv} \beta \rho \, d\forall + \iint_{c\cdot s} \beta \rho \vec{V} \cdot \vec{dA}$$

$\Rightarrow \frac{dB}{dt}$ = Time rate of change of water vapor in the atmosphere

$$\Rightarrow \boxed{\frac{dB}{dt} = \dot{m}_v}$$

$\dot{m}_v$ = Mass flow rate of water vapour.

$\dot{m}_v$ = (+ve) for evaporation, (−ve) for condensation.

$$\Rightarrow \boxed{m_v = \frac{d}{dt}\iiint_{C.V} q_v \rho_a \, d\forall + \iint_{C.S} q_v \rho_a \vec{V}\cdot d\vec{A}}$$

(*) Vapor Pressure :- (e) :- The partial pressure exerted by the water vapour in the atmosphere.

The partial pressure is independent of presence of other gases

↳ Dalton's Law ⇒ Ideal gas Law

(*) $\boxed{e = \rho_v \cdot R_v \cdot T}$

ρ_v = Density of vapour

R_v = ⟶ gas constant for vapour

T = Absolute Temperature (K)

Here $\boxed{R_v = \frac{R_{univ}}{M_v}}$

$\Rightarrow \boxed{e = \rho_v R_v T}$ ⟶ Exerted by vapour only

P = Total pressure

⇒ $(P-e)$ = Partial pressure of dry air

$\Rightarrow \boxed{(P-e) = \rho_d R_d T}$

R_d = 287 J/kg·K

$$\boxed{P_a = P_d + P_v}$$

$$\Rightarrow \quad P_d = P_a\left(1 - \frac{P_v}{P_a}\right)$$

$$\Rightarrow \quad \boxed{P_d = P_a(1 - q_v)}$$

$$\Rightarrow \quad \boxed{R_v = \frac{R_d}{0.622}}$$

combine

$$\Rightarrow \quad \boxed{q_v \approx 0.622\,\frac{e}{P}}$$

e = Pressure exerted by Vapour (Vapor pressure)

P = Pressure exerted by moist air

$$\boxed{R_a = R_d(1 + 0.608\,q_v)}$$

Here $R_d = 287\ \mathrm{J/kg\cdot K}$

Saturation vapor pressure (e_s) → It is the max pressure exerted by the water vapour in the air, for a given temperature.

For a given temp, the pressure exerted by the for highest possible moist air.

$$e_s = 611 \exp\left(\frac{17.27\,T}{237.3 + T}\right)$$

(Pascals)

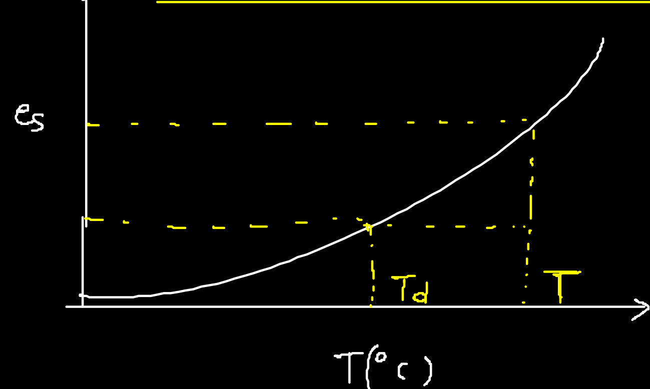
1 Pa = 1 N/m²

T in °C

e_s v/s T

Saturation vapor pressure.

Saturation vapor pressure: Partial pressure of water vapor when the air is completely saturated



Δ = slope of e_s v/s T curve $\Rightarrow \frac{de_s}{dT}$

$$\Rightarrow \boxed{\Delta = \frac{4098\, e_s}{(237.3 + T)^2}}$$

$\Rightarrow$ Units:- $\boxed{Pa/°C}$

Not 273, NOT Kelvin → °C conversion.

Relative Humidity

↳ Ratio of vapor pressure to saturation vapor pressure.

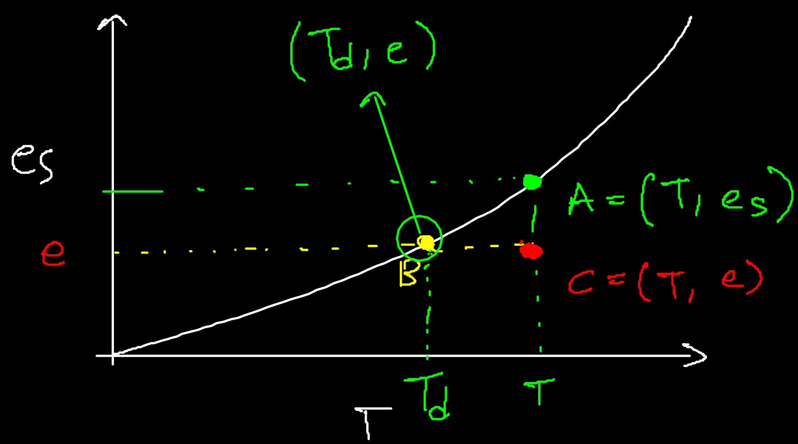
$$\Rightarrow \boxed{R_h = e/e_s}$$

Dew point Temperature (T_d)

The temperature at which air would just become saturated when cooled at constant pressure and moisture content

⊛ T_d is the temperature at which the air would just become saturated at a given specific humidity (or) moisture level

Ⓐ T_d is actually a measure of humidity in the atm.

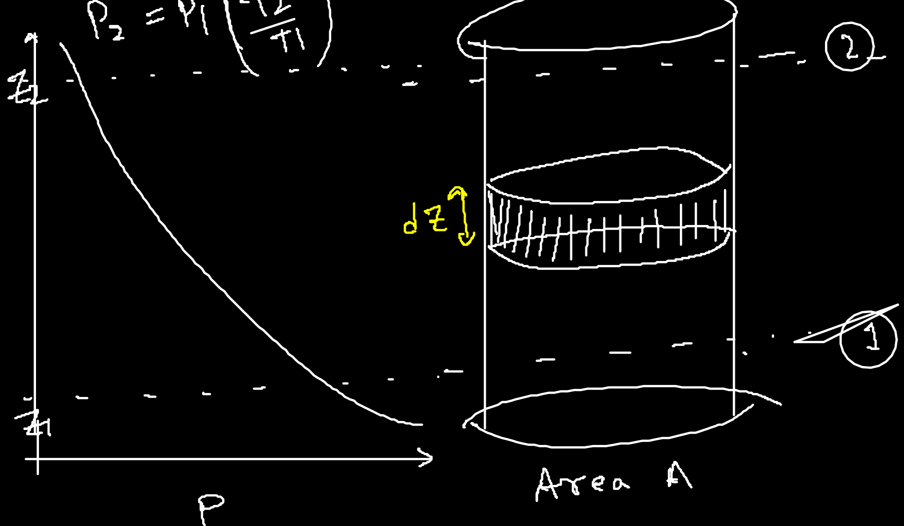


⊛ T_d is temp. at which the air is saturated for vapour pressure e.

⋆ For e to become e_{sat} → what is the $\boxed{\text{Temp} = T_d}$

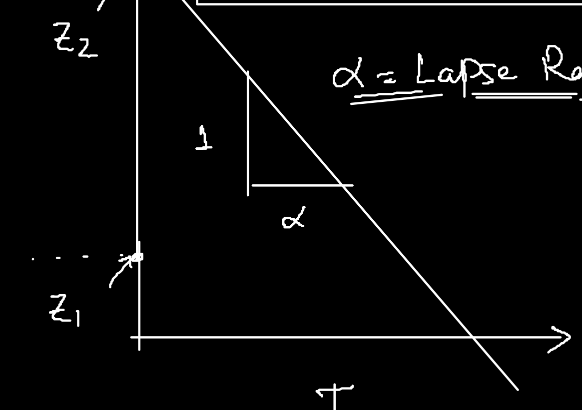
Water vapor in a static column

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{(g/\alpha R_a)}$$



$$\boxed{T_2 = T_1 - \alpha(z_2 - z_1)}$$

α = Lapse Rate



Ⓐ We need two equations → Ideal gas Law
→ Hydrostatic pressure Law.

* Ideal gas law = $P = \rho_a R_a T$

* Temp drops with altitude. $\boxed{\frac{dT}{dZ} = -\alpha}$

* For Dry air: $\alpha = 9.8°C/km$

Saturated adiabatic $\alpha = 6.5°C/km$.

Hydrostatic law

$$\frac{dP}{dZ} = -\rho_a g$$

* $\Rightarrow$ $P = \rho_a R_a T \Rightarrow \rho_a = \left(\frac{P}{R_a T}\right)$

$$\Rightarrow \frac{dP}{dZ} = -\frac{P}{R_a T} \cdot g$$

But $dT = -\alpha dZ$

$\Rightarrow dZ = \left(\frac{dT}{-\alpha}\right)$

$$\Rightarrow \frac{dP}{P} = -\frac{g}{R_a T} \cdot dZ$$

$$\Rightarrow \frac{dP}{P} = -\frac{g}{R_a T} \cdot \frac{dT}{-\alpha}.$$

$$\Rightarrow \frac{dP}{P} = \frac{g}{\alpha R_a T} \cdot dT$$

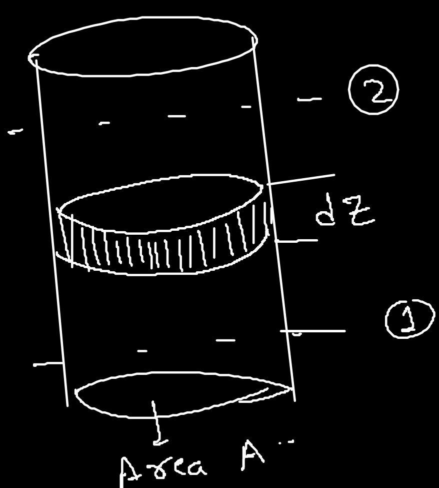
$$\Rightarrow \int_{P_1}^{P_2} \frac{dP}{P} = \frac{g}{\alpha R_a} \cdot \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\Rightarrow \ln\left(\frac{P_2}{P_1}\right) = \frac{g}{\alpha R_a} \cdot \ln\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow \quad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}}$$

$$\Rightarrow \quad \boxed{P_2 = P_1 \left(T_2/T_1\right)^{g/\alpha R_a}}$$

Now



Precipitable water: The amount of moisture in the atmospheric column at any given time.

Mass of moist air in the c.v

$$= (\rho_a \times A \times dZ)$$

(A) mass of water vapour in the c.v $\Rightarrow \rho_v \times A \times dz$.

$$\boxed{q_v = \frac{\rho_v}{\rho_a}}$$

$$\Rightarrow \text{Mass of water vapor} = (q_v \rho_a \cdot A \cdot dZ)$$

$$\Rightarrow \text{Total Precipitable mass} = \int_{Z_1}^{Z_2} m = m_p$$

$$\Rightarrow \quad TPW = \boxed{m_p = \int_{Z_1}^{Z_2} q_v \rho_a A \, dZ}$$

(or)

$$\boxed{m_p = \sum_{i=1}^{n} \left(\bar{\rho}_{a_i} \cdot \bar{q}_{v_i} \times A \times \Delta z\right)}$$

↳ Discrete form

$\bar{\rho}_{a_i}$ = Avg. air density over interval 'i'

$\bar{q}_{v_i}$ = Avg. q_v over the interval i

$$\boxed{\text{Equivalent Rainfall depth} = \frac{m_p}{\rho_w A}}$$

$z_2, T_2, P_2, \rho_{a_2}, q_{v_2}$

$z_1, T_1, P_1, \rho_{a_1}, q_{v_1}$

$$\bar{q}_v = \left(\frac{q_{v_1}+q_{v_2}}{2}\right)$$

$$\bar{\rho}_a = \left(\frac{\rho_{a_1}+\rho_{a_2}}{2}\right)$$

Here $q_{v_1} = 0.622 \dfrac{e_{s_1}}{P_1}$

$e_{s_1} = 611\exp\left(\frac{17.\ldots}{\ldots}\right)$

↳ At each height, we can find.

$q_{v_2} = 0.622 \dfrac{e_{s_2}}{P_2}$

(*) We assume that $\boxed{R_a \simeq R_d}$

↳ Variation of R_a with q_v is negligible

Formation of Precipitation

(*) Two basic mechanisms → Lifting of moist air mass.

→ Nucleation [Availability of sites to form water droplets].

(*) Frontal lifting of air mass:- Warm air is lifted over the cooler air by the frontal action. ⇒ $\boxed{\text{Frontal Precipitation}}$

⊛ Orographic:- The air rises due to obstruction by topography.
↳ $\boxed{\text{Orographic precipitation}}$

⊛ Convective:- When the air mass is drawn upwards by the convective action, due to temperature gradient
↳ $\boxed{\text{Convective precipitation}}$

⊛ Nucleation:- Sites are required to form water droplets.
(Aerosols (or) Nucleus) ⇒ Dust particles which are ionized
⇒ They attract the water.

⇒

⊛ Forms of precipitation →

↳ Rain. (size: 0.5mm – 6mm)

0 – 2.5 mm/hr ⇒ Light Rain
2.5 – 7.5 ⇒ Moderate
> 7.5 mm/hr ⇒ Heavy Rain

Drizzle (< 0.5mm size
i < 1mm/hr)
Terminal velocity is very low
(Appears to float).

→ Glaze:- Ice coating generally smooth, formed on exposed surfaces by the freezing of super cooled water
Sp. gr = 0.8 to 0.9

⊛ Rime:- White opaque deposit of ice-granules seperated by the trapped air, formed by rapid freezing of supercooled water drops impinging on exposed objects. (SG = 0.2 to 0.3)

Snow:- Consists of ice crystals which falls in solid form.
⊛ They combine to form snowflakes.

Hail:- formation of ice balls of various shapes and sizes.

Size ≥ 5mm. Largest hailstorm recorded :- Coffeyville Kansas (3/9/1970) → sizes ≈ circumference ≈ $\boxed{44 cm}$

Sleet:- Transparent ice / solid ice formed by freezing of raindrops near to the earth surface.

Q:? In the sub freezing temperature, in a mixture of ice crystals, and liquid-water droplets, evaporation of water droplets takes place & the evaporated water then condenses on ice crystals

↳ Ice crystals grow in size - to become snow/hail. WHY?

(A) The e_{sat} over ice is lesser than over the liquid water.

It is because the temperature over ice is less.

In India → SW Monsoon (JJAS)

→ Transition - I; Post monsoon (Oct - Nov)

→ Winter. (Dec - Feb)

→ Transition II; (Mar - May)

SOUTHWEST MONSOON

(A) Runs from June to September

↳ 75% of avg. annual RF is recieved during this time

↳ Highly uneven in space and time.

* Rainiest month ⇒ July.

* This monsoon is accomp. by SW wind ≈ 30–70 kmph.

* Monsoon originates in the Indian ocean, appears first in Kerala by the end of May.

Monsoon winds

Arabian Sea Branch

- Starts in Kerala @ 1^{st} week of June
- Goes to North – Karnataka, → Maharashtra → Delhi → Around 4th week.

Bay of Bengal branch

* Starts @ Assam @ 1^{st} week June

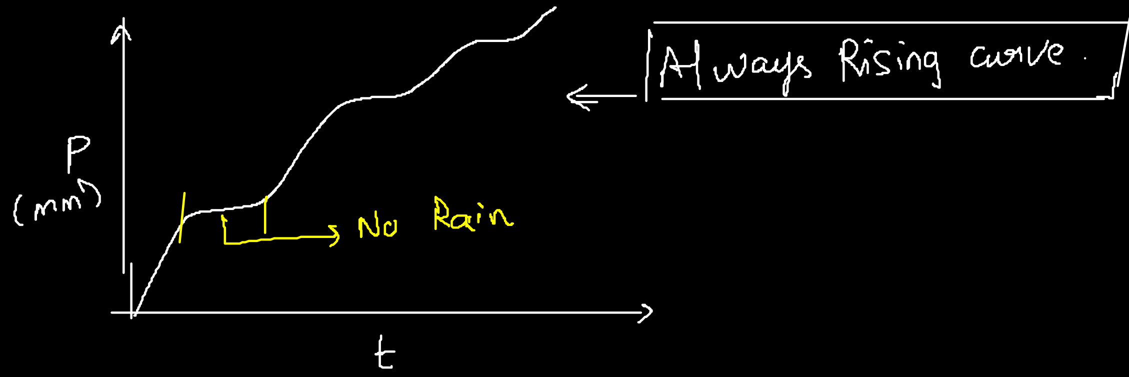
NE region → Bihar → UP → Delhi → 4^{th} week of June

* Combine in Central India to form $\boxed{\text{Monsoon Trough}}$

* The Monsoon trough oscillates over the country which dictates the rainfall. → The MT is low pressure region formed by combining of two winds.

* $\boxed{\text{Withdrawl of Monsoon}}$ → Substantial Rainfall Activity starts September in North India.

Rainfall Mass curve :- Cumulative curve of rainfall depth



(A) Maximum Depth Duration Curve

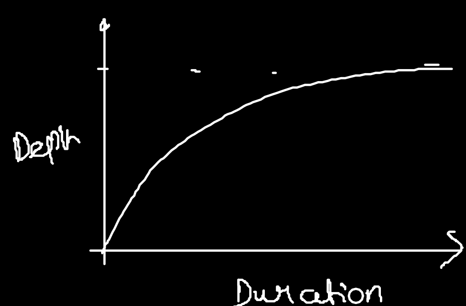
- ↳ useful in hydrologic design
- ↳ Gives info on max RF depth that occur in given duration.

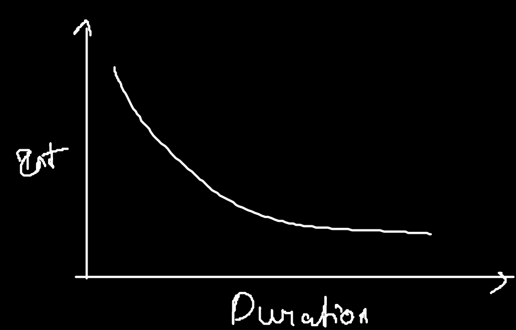
→ Develop the RF mass curve using data of most severe storm.

→ Select desirable durations

→ For each duration, compute max RF depth by subtracting cumulative rainfall depths b/w selected time durations.

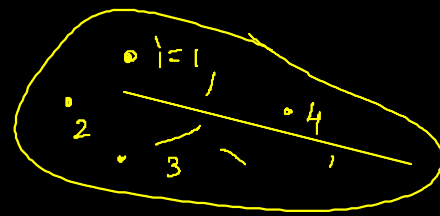
→ Repeat this for all durations.





Ⓐ Average Area Rainfall.

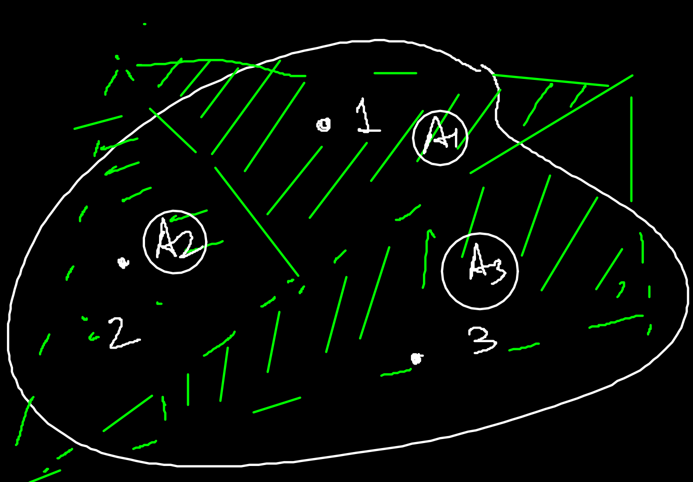
$$\boxed{\bar{P} = \frac{1}{N}\sum_{i=1}^{N} P_i}$$



Ⓐ Rarely used ⇒ Not able to account for spatial coverage of the raingauge.

Ⓐ Thiessen Polygon :- Assumes that (i) At any point in a catchment, the rainfall is same as that at the nearest raingauge.

② The RF depth recorded at a raingauge is applied out to a distance halfway to the next station. in any direction.



$$\boxed{\bar{P} = \frac{\sum_{i=1}^{n} A_i P_i}{\sum A_i}}$$

* Area represented by i^{th} raingauge.

Ⓐ Drawbacks:- (i) Not flexible as a new theissen network must be constructed for every additional raingauge.

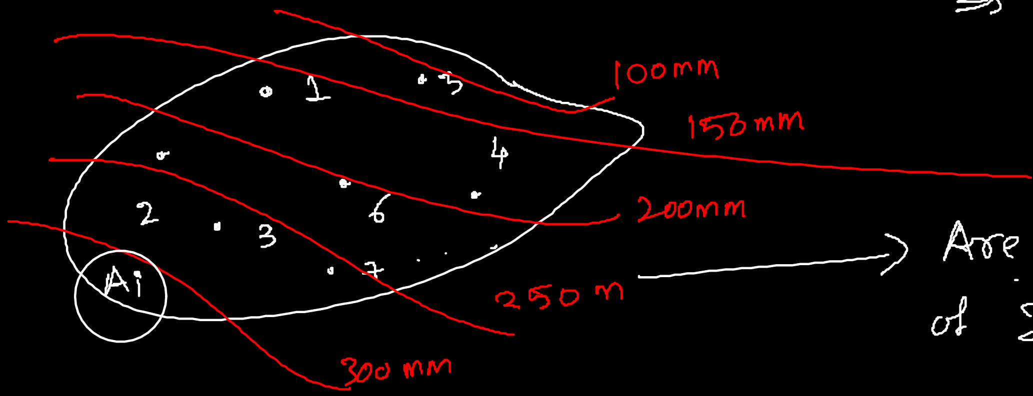
② Does not account for orographic influences

③ Effect of RF magnitude itself is not taken into consideration.

(*) Isohyetal method → overcomes above limitations.

(*) Isohyet is a line joining equal RF areas.

⇒ [Contours of equal rainfall depth.]



→ Area (Ai) b/w each pair of Isohyets

$$\boxed{\bar{P} = \frac{1}{A} \cdot \sum P_i A_i}$$

P_i = Avg. RF depth for the two boundaries

(4) Reciprocal Method

→ Effect of RF on a gauge point on any other point in the catchment inversely prop. distance b/w them.

$X_1(x_1, y_1)$; $X_2(x_2, y_2)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\boxed{\bar{P} = \frac{\sum P_i W_i}{\sum W_i}}$$

(*) $\boxed{\text{Weightage} = \frac{1}{D}}$.

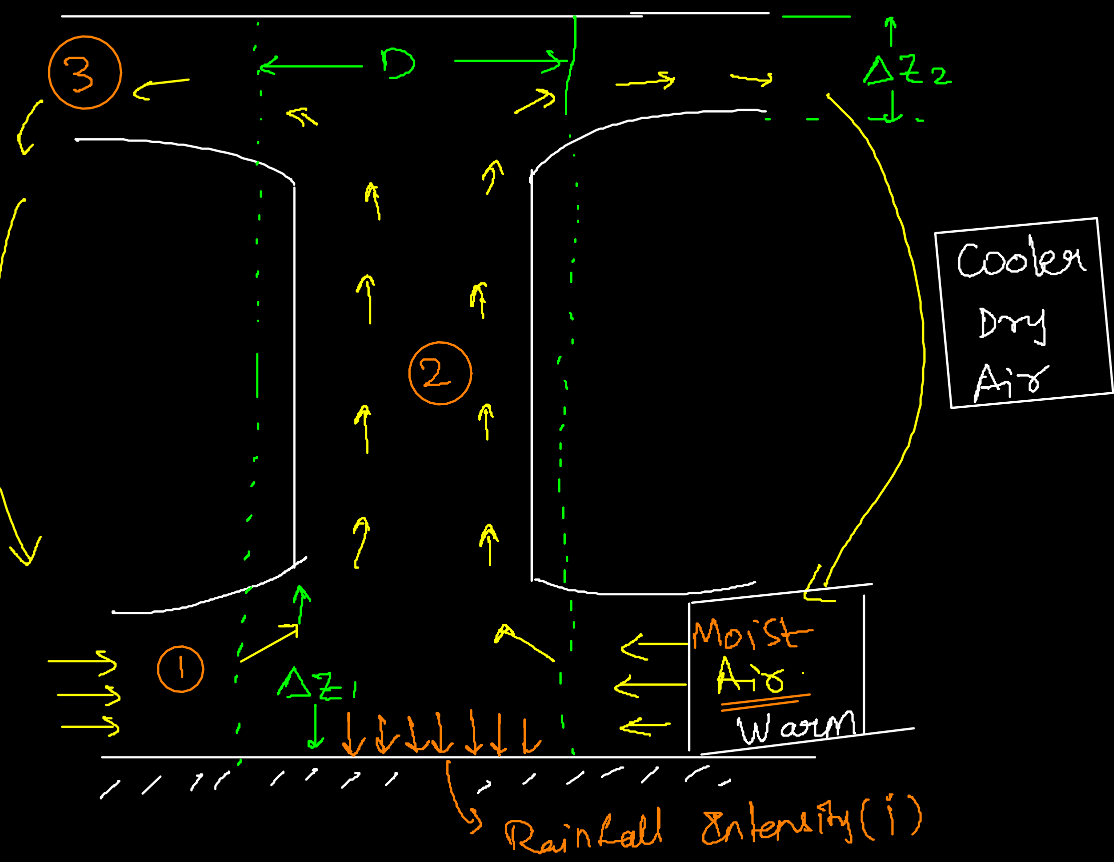
THUNDERSTORM CELL MODEL

(*) A vertical column of atmosphere
- → Inflow Region → (1)
- → uplift Region → (2)
- → out flow Region → (3)

⊛ Convective cell

This air descends over wide area, pick up more moisture and reenter into the cell close to the ground.

Convective circulation.



⊛ Continuity :

$B =$ Mass of water vapour

$$\boxed{\beta = q_v}$$

$$\dot{m}_v = \frac{d}{dt}\iiint_{c.v} q_v \rho_a \, d\forall + \iint_{c.s} q_v \rho_a \vec{V}\cdot d\vec{A}$$

At inflow : $P_1, \rho_1, V_1, q_{v_1}, T_1$

At outflow : $P_2, \rho_2, V_2, q_{v_2}, T_2$

$\dot{m}_v =$ Time rate of change of mass

$\dot{m}_v =$ Mass/time

Assume precipitation intensity $= i$, falling over A

$$\Rightarrow \dot{m}_v = -\frac{\rho_w \times A \times \text{depth}}{\text{Time}} = \boxed{-\rho_w A i = \dot{m}_v}$$

$\Rightarrow$ Now, flow is steady

$$-\rho_w A_i = \iint_{c \cdot s} q_v \cdot \rho_a \vec{V} \cdot \vec{dA}$$

$$\Rightarrow -\rho_w A_i = \iint_{③} q_v \rho_a \vec{V} \cdot \vec{dA} - \iint_{①} q_v \rho_a \cdot \vec{V} \cdot \vec{dA}$$

$$\Rightarrow -\rho_w A_i = (q_{v_2} \cdot \rho_{a_2} \cdot \vec{V_2}) \cdot \pi D \Delta z_2 - (q_{v_1} \rho_{a_1} \cdot \vec{V_1}) \cdot \pi D \Delta z_1$$

①

④ Consider dry air; β = Mass of dry air.

$\beta = \dfrac{\rho_d}{\rho_a}$ ∴ For steady state

$$\Rightarrow \frac{dB}{dt} = \iint_{c \cdot s} \beta \rho \vec{V} \cdot \vec{dA} = \iint_{c \cdot s} \frac{\rho_d}{\rho_a} \cdot \rho_a \vec{V} \cdot \vec{dA}$$

$$\Rightarrow \boxed{\frac{dB}{dt} = \iint_{c \cdot s} \rho_d \cdot \vec{V} \cdot \vec{dA}}$$

Here, the dry air content does not change $= \dfrac{dB}{dt} = 0$

$$\Rightarrow 0 = \iint_{c \cdot s} \rho_d \vec{V} \cdot \vec{dA} = \iint_{②} \rho_d \vec{V} \cdot \vec{dA} - \iint_{①} \rho_d \cdot \vec{V} \cdot \vec{dA}$$

But $\boxed{\rho_d = \rho_a (1 - q_v)}$

⇒ Simplifying both Eqⁿs.

$$\boxed{i = \frac{4 \rho_{a1} \cdot \vec{V_1} \cdot \Delta Z}{\rho_w D} \left(\frac{q_{v1} - q_{v2}}{1 - q_{v1}} \right)}$$

Mass flow Rate:-

$$\boxed{m_p = \rho_{a1} \cdot \vec{V_1} \, (\pi D \Delta Z_1) \left(\frac{q_{v1} - q_{v2}}{1 - q_{v2}} \right)}$$