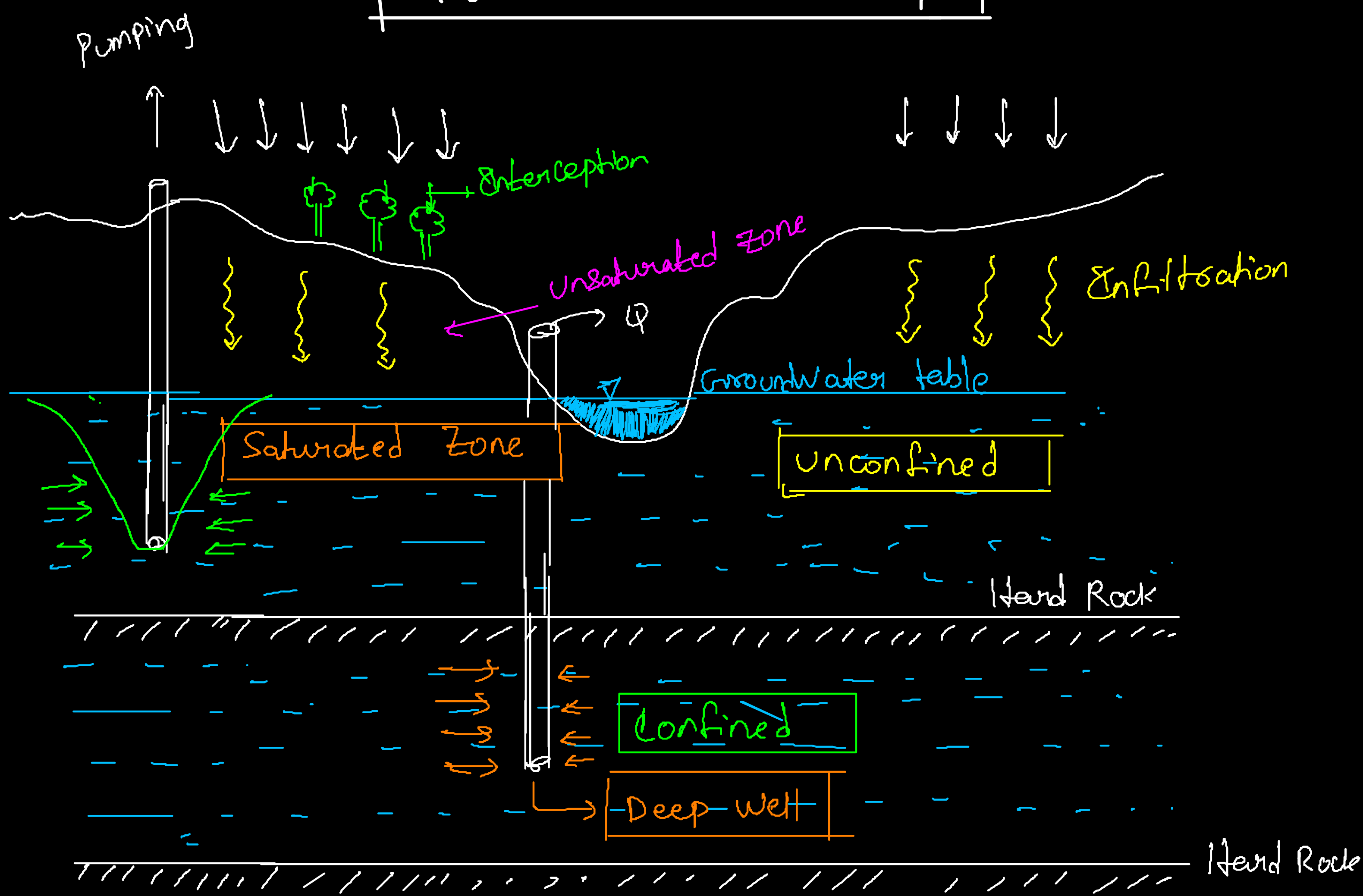
GROUNDWATER HYDROLOGY



AQUIFER:- An aquifer is a geological formation that contains significant amount of water in the saturated zone;

→ It can yield significant amounts of water.

→ It can store and yield water —
- → store water
- → Transmit water.

④ The unsaturated zone of the permeable material is a part of the aquifer

→ Uncosolidated sands and gravels.

→ Also called Groundwater reservoirs / water bearing formation.

⊛ AQUICLUDE :- It can store water but it's relatively having low permeability → It does not yield water Ex:- Clay

⊛ AQUIFUGE :- A relatively impermeable formation that can Neither contain nor transmit water ; Ex:- Granites Rock etc.

⊛ AQUITARD :- It retards the movement of water. It is Saturated but Poorly Permeable → It retards the movement of groundwater and does not yield water freely to wells.

→ It may transmit water to and from adjacent aquifers

AQUIFER PROPERTIES

(i) Voids → The portion of a rock/soil not occupied by the solids can be occupied by air or water. These empty spaces are called Voids (interstices/ Pores/ Porespaces)

↳ The Voids serve as conduits for the water movement.

↳ We characterize them by size, shape, irregularity etc.

→ 2 types of Voids
- → original (Primary)
- → Secondary voids.

→ **Primary Voids**:- The ones that were created originally by the geological processes, governing the origin of the soil / Rock.

↳ Found in Sedimentary & Igneous Rocks.

→ Secondary Voids → The ones that are developed after the formation of rocks. → Joints / fractures etc; openings created by plants / Animals

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POROSITY :- Ratio of volume of voids to the total volume.

$$\boxed{\eta = \frac{V_v}{V_t}}$$

$$\boxed{\text{Void Ratio } (e)} = \left(\frac{V_v}{V_s}\right) \rightarrow \frac{\text{Volume of Voids}}{\text{Volume of Solids}}$$

→ The porosity (η) can also be given as

$$\boxed{\eta = \frac{\rho_m - \rho_d}{\rho_m} = 1 - \frac{\rho_d}{\rho_m}}$$

ρ_m = Density of solids $= \left(\frac{\text{Dry wt}}{V_s}\right)$

$$\rho_d = \text{Bulk density} = \frac{\text{Dry wt.}}{\text{Dried vol}} = \boxed{\frac{W_t}{V_t}}$$

$$\text{Volumetric water content } (\theta) = \frac{\text{Vol. of water}}{\text{Total Volume}} = \left(\frac{\theta_v}{V_T}\right)$$

→ If we define the water content by weight.

Gravimetric water content : θ_w = Ratio of weight of water / Total weight of dried Sample

$$\boxed{\theta_w = \frac{W_t - W_d}{W_d}}$$

→ Percent saturation → Percentage of voids filled by water.

$$\boxed{\% \, S_r = \frac{\theta_v}{\eta}}$$

→ Effective Porosity → (S_e) → Actual available porosity accounting for the hygroscopic water

$$\boxed{S_e = \eta - \theta_r}$$

θ_r = Residual M.C

↳ Can't remove by drainage

→ Porosity varies with depth

$$\boxed{\eta_z = \eta_0 e^{-az}}$$

η_z = Porosity at depth Z

η_0 = Porosity at surface

a = constant

Groundwater Movement

The groundwater movement in $\boxed{\text{Saturated Zone}}$ is governed by the $\boxed{\text{Darcy's Law.}}$

→ The flow rate through Porous media is proportional to head loss and also inversely proportional to length of the flow path.

$$\boxed{q \propto h_f} \quad \text{and} \quad \boxed{q \propto \frac{1}{L}}$$

$$\Rightarrow \boxed{q = \frac{-k\,h_f}{L}}$$

q = Darcy's flux (L^2/T) (m^2/s)
↳ Flow per unit length

h_f = head loss b/w 2 location situated 'L' m apart

k = Hydraulic Conductivity (L/T) (m/s)

$$\boxed{Q = \frac{-KAh_f}{L}} = \boxed{-KA\left(\frac{dh}{dL}\right)}$$

⇒ $q \approx V \approx$ Darcy's velocity ; If we consider $\left[i = -(dh/dL)\right]$

$$\Rightarrow \boxed{V = ki}$$

i = Hydraulic gradient

→ The actual flow velocity is higher than the Darcy velocity.

$$\boxed{V_a = V/\eta}$$

η = Porosity of the Soil.

$$\Rightarrow \boxed{Q_A = \frac{Q}{A_p}}$$

A_p = Actual area available for the water to pass

Validity of Darcy's Law

(i) Valid only for Laminar flow.

(2) Poisulle's law is valid ⇒ Small tiny pores

(3) Reynolds Number = $\frac{\text{Inertial force}}{\text{Viscous force}}$ = Re valid for < 1 ⇒ Darcy Law applicable

$(1-10)$ → Applicable, but no accurate

→ As the inertial forces increase, turbulence increases → R_e increases.

→ In most GW flows → $\boxed{R_e \leq 1 \rightarrow \text{Darcy Law valid}}$ → when the gradient is steep then we can't use

→ During pumping, we can't apply this equation.

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* **Hydraulic Conductivity :- (K) :-** Ability of a soil to allow for movement of water.

$$\boxed{K = f\,(\text{soil type, Liquid})}$$

→ $\boxed{K \propto d_m^2}$ d_m = mean dia. of Soil

→ $\boxed{K \propto \gamma}$ γ = wt density

→ $\boxed{K \propto \frac{1}{\mu}}$ → μ = Dynamic Viscosity

$$\Rightarrow K = \frac{C \cdot d_m^2 \gamma}{\mu}$$

$$\Rightarrow \boxed{K = C d_m^2 \cdot \frac{g}{\nu}}$$

* **Intrinsic Permeability**: The component of permeability (K) dependent on soil properties only is $\boxed{K_s}$

$$\Rightarrow \boxed{K_s = C d_m^2}$$

Units:- $K_s = m^2$ (or) Darcy $= 9.87 \times 10^{-13} m^2$

* **Transmissibility** (T) ⇒ The rate at which water is transferred through a unit width under unit hydraulic gradient

$$\boxed{T = KB}$$

Determination of Hydraulic Conductivity & Transmissibility

→ Using formulae

→ Laboratory → Falling head
→ Constant head

→ Tracer test

→ Auger test

→ Pumping test

AQUIFER Properties

- **Specific Retention (S_r)** → The volume of water that the aquifer can retain against the force of gravity divided by the total volume.

$$\boxed{S_r = \frac{V_{Retained}}{V_{total}}}$$

V_r = Volume occupied by the retained water → Cannot be drained by gravity and we cannot pump it out of the aquifer

- **Specific Yield** (S_y) :→ It is the ratio of volume of water, that after saturation, can be drained by gravity, to it's own volume.

$$\boxed{S_y = \frac{V_{yielded}}{V_{total}}}$$

V_{yield} = Total volume that can be drained by gravity and pumping

$$V_y + V_R = V_{voids}$$

$$\Rightarrow \frac{V_y}{V_t} + \frac{V_R}{V_t} = \frac{V_{voids}}{V_t} \Rightarrow \boxed{S_y + S_R = \eta}$$

→ S_y → depends on
- → Grain size and shape → Fine grain → Less yield
- → Distribution of pores
- → Compaction of the stratum
- → Time of drainage → Generally decreases with time.
- → Yield decreases with depth ← B/c of higher compaction.

≈ 27% for coarse sand
≈ 3% for clays.
≈ 44% for peat

- **Ex:-** find the avg drawdown over an area where 25 Mm³ was pumped. The area = 150 km². S_y = 25% (unconfined);

(A) $S_y = \dfrac{\text{Volume drained}}{\text{Total volume}} \Rightarrow \dfrac{25\ \text{Mm}^3}{A \times (b)} = S_y$

$$\Rightarrow A \times b = \frac{25\ \text{Mm}^3}{0.25} \Rightarrow b = \frac{25 \times 10^6\ m^3}{150 \times 10^6 \times 0.25}$$

$$\Rightarrow \boxed{b \simeq 0.67\,m}$$

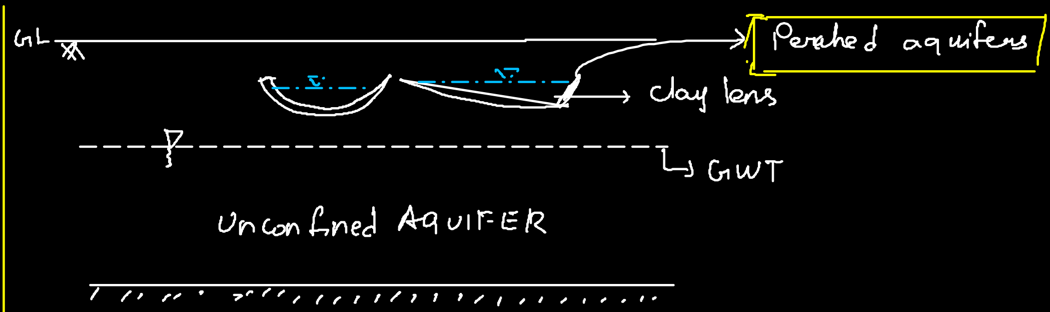
TYPES OF AQUIFERS

① **Unconfined AQUIFERS** :- It is not confined b/w two layers. It's the one in which the GWT is undulating in the form and slope, depending on recharge and discharge from wells.

→ The GWT is under atmospheric pressure
→ The Rise and fall of GWT correspond to change in aquifer storage.
→ The contour maps of water table in the wells help in determining the quantities of water available, distribution and movement.

④ **Perched aquifer** :- Special case of unconfined aquifer due to impermeable strata in the unsaturated zone.

→ Low yield wells
→ Shallow water levels
→ Ex:- Clay lens in Sedimentary deposits



Confined aquifers → Artesian well, Pressure well

→ The water is at a pressure greater than atmospheric
→ Water is confined between two impermeable strata.
→ If we dig a well, the water will rise above the upper confined layer
→ Water enters the confined aquifer in regions where the confining layer meets ground.
→ When the confining bed ends subsurface → Confined aquifer becomes unconfined
→ Recharge comes from far regions or due to leakage from upper confining layer
⇒ The rise and fall of water level in a confined well is due to changes in pressure. **NOT** due to changes in Storage. → Serve primarily as conduits for transmitting the water from recharge areas to natural / artificial discharge locations.
⇒ The water from confined aquifer can reach ground level at some gravity springs / lakes
→ The contours of well levels in a confined aquifer gives the piezometric levels.

(R) Piezometric surface / Potentiometric surface :- An imaginary surface coinciding with the hydrostatic pressure level of the water in the confined aquifer.

→ The water level of a well in confined aquifer = Piezometric surface.

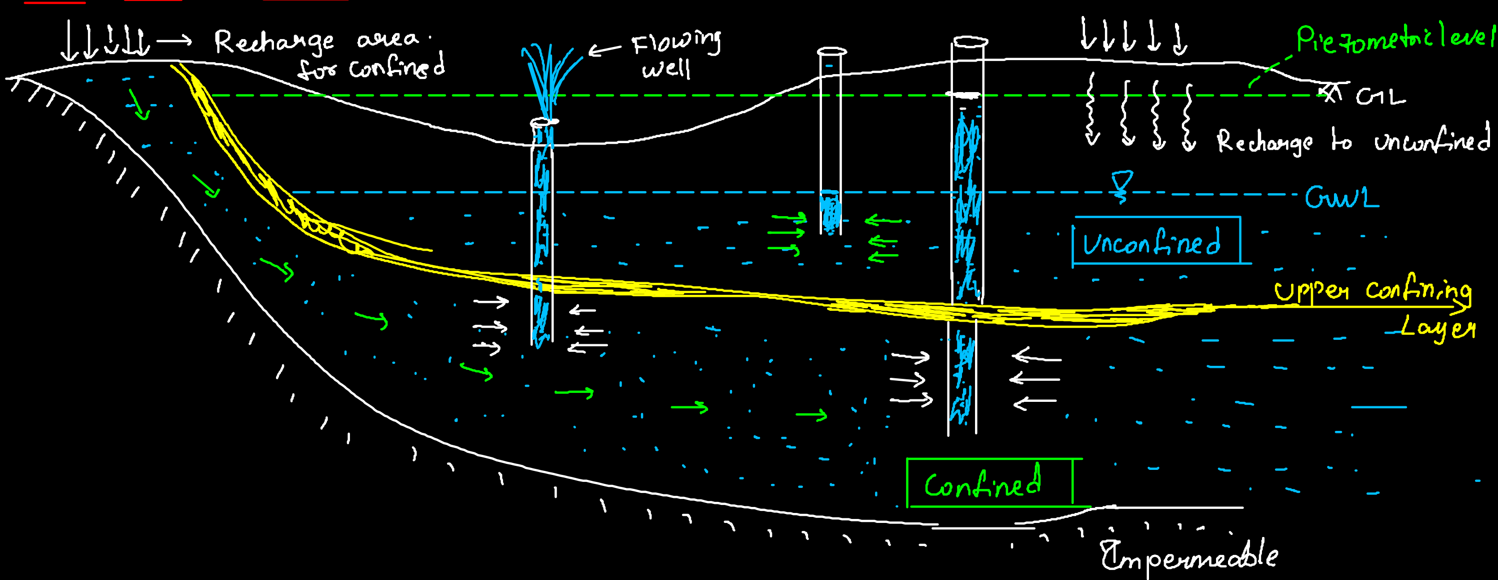
→ The piezometric surface may be above / below the ground level

→ Contour maps of the piezometric surface can be prepared from water levels.

+ Flowing wells
+ Springs.

→ If piezometric level falls below the upper confining layer → becomes unconfined

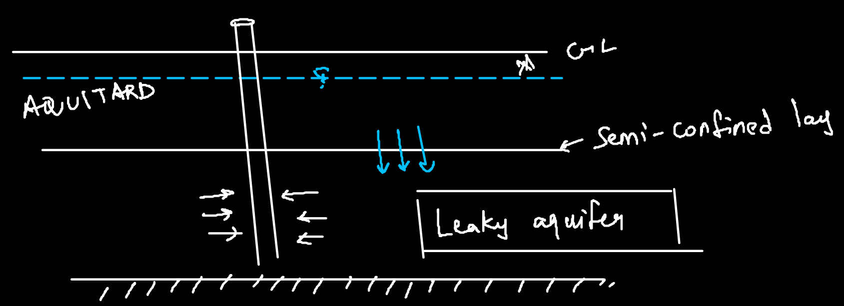
→ Usually, the unconfined aquifer occurs above a confined aquifer



→ Leaky aquifer :- Also called Semi-confined. → They occur more frequently

→ When a permeable strata is overlain (or) underlain by a semi-confining layer such as an aquitard.

→ Water from leaky aquifer is yielded by
- → Horizontal flow
- → Vertical flow through aquitard into the leaky aquifer.

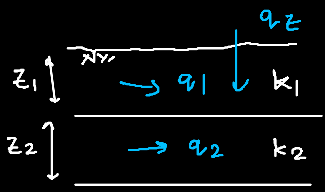


Idealized AQUIFER

- Generally, aquifers are heterogenous and anisotropic.
- So, we use an Idealized aquifer which is homogenous and isotropic.
- They do not exist in nature but good / practical quantitative approximations can be obtained.

Anisotropic aquifer:- The aquifer properties are different along different directions.

→ Let k_x and k_z be the horizontal effective hydraulic conductivity in the horizontal and vertical direction.



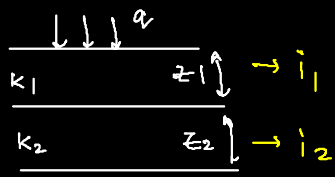
→ When the flow is horizontal

$$q = q_1 + q_2 \implies q = k_1 i z_1 + k_2 i z_2$$

and the gradient : $i = i_1 = i_2$ $\implies k_x i (z_1 + z_2) = i(k_1 z_1 + k_2 z_2)$

$$\implies \boxed{k_x = \frac{k_1 z_1 + k_2 z_2}{z_1 + z_2}}$$ ← effective hydraulic conductivity in the x-direction.

Case 2 :- When flow is across the layers.



→ Here, the hydraulic gradient varies as i_1 and i_2 in the layers. and $q = q_1 = q_2$

$$\implies q = q_1 = q_2 \quad \text{and } i = i_1 + i_2$$

$q = k \cdot \left(\frac{z}{L}\right)$

$L = z_1 + z_2$

$$\implies \frac{q(z_1+z_2)}{k_z} = \frac{q(z_1)}{k_1} + \frac{q z_2}{k_2}$$

$$\implies \frac{z_1+z_2}{k_z} = \frac{z_1}{k_1} + \frac{z_2}{k_2} \implies$$

$$\implies \frac{1}{k_z} = \left(\frac{z_1}{k_1} + \frac{z_2}{k_2}\right)\frac{1}{z_1+z_2}$$

$$\implies \boxed{k_z = \frac{z_1 + z_2}{\frac{z_1}{k_1} + \frac{z_2}{k_2}}}$$

Note:- Normally $\boxed{k_x > k_z}$ in various aquifer $\implies \boxed{\frac{k_x}{k_z} \approx 2 \text{ to } 10}$

⇒ Then we can apply Darcy's law:-

$$\boxed{\begin{aligned} &\rightarrow \text{Horizontal}: \quad V_x = k_x i_x \\ &\rightarrow \text{Vertical}: \quad V_z = k_z i_z \end{aligned}}$$

* If the flow is at an angle β with the horizontal

$$\boxed{V_\beta = K_\beta \cdot Z_\beta}$$

$$\boxed{k_\beta = \frac{\cos^2\beta}{k_x} + \frac{\sin^2\beta}{k_z}}$$

Flow equations

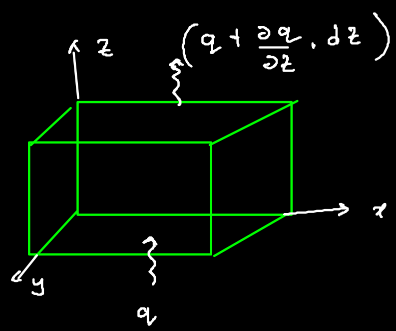
→ Saturated, Steady flow in confined aquifer

→ Use the RTT
$$\frac{dB}{dt} = \frac{d}{dt}\iiint_{C.V} \beta \rho \, d\forall + \iint_{C.S} \beta \rho \, \vec{V}\cdot d\vec{A}$$

Here B = mass ⇒ $\beta = \frac{dm}{dm} = 1$

$$\Rightarrow \boxed{\frac{dB}{dt} = \frac{d}{dt}\iiint_{C.V} \rho \, d\forall + \iint_{C.S} \rho \vec{V}\cdot d\vec{A}}$$

⇒ Consider the control volume ⇒ (dx dy dz)



$$\Rightarrow \frac{d}{dt}\iiint_{C.V} \rho \, d\forall = \frac{\partial \rho}{\partial t}(dx\,dy\,dz)$$

$$\Rightarrow \iint_{C.S} \rho \vec{V}\cdot d\vec{A} = \left[\rho\left(q + \frac{\partial q}{\partial z}.dz\right)dx\,dy - q \cdot dx\,dy\right]$$

$$= \iint_{C.S} \rho \vec{V}\, d\vec{A} = \left[\frac{\partial q}{\partial z}.dx\,dy\,dz\right] = \frac{\partial(\rho q)}{\partial z} dx\,dy\,dz.$$

⇒ So, the Continuity Eq^n. in Z-dir^n

$$\frac{dB}{dt} = \frac{\partial \rho}{\partial t}(dx\,dy\,dz) + \frac{\partial(\rho q)}{\partial z} dx\,dy\,dz.$$

$q_x = V_x$ (Darcy velocity)

→ for mass conservation ⇒ $\frac{dB}{dt} = 0$

$$\Rightarrow \boxed{\frac{\partial(\rho q_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0}$$

For Compressible and Unsteady

→ For all 3 dir^n

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho q_x)}{\partial x} + \frac{\partial(\rho q_y)}{\partial y} + \frac{\partial(\rho q_z)}{\partial z} = 0}$$

For incompressible Steady state ⇒

$$\boxed{\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial z} + \frac{\partial q_z}{\partial z} = 0}$$

⇒ Now, apply Darcy's Law ⇒ $q_x = k_x \frac{\partial h}{\partial x}$; $q_y = k_y \cdot \frac{\partial h}{\partial y}$; $q_z = k_z \cdot \frac{\partial h}{\partial z}$.

$$\Rightarrow \frac{\partial}{\partial x}\left(k_x \cdot \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \cdot \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \cdot \frac{\partial h}{\partial z}\right) = 0$$

$$\Rightarrow k_x \cdot \frac{\partial^2 h}{\partial x^2} + k_y \cdot \frac{\partial^2 h}{\partial y^2} + k_z \cdot \frac{\partial^2 h}{\partial z^2} = 0$$

If Isotropic ⇒ $k_x = k_y = k_z$

* Steady state
* Homogenous
* Isotropic

$$\Rightarrow \boxed{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0} \Rightarrow \boxed{\nabla^2 h = 0} \Leftarrow \text{Laplace Eq}^n$$

⇒ Unsteady Saturated flow → Confined AQUIFER → Compressibility effects have to be considered.

The Governing Eq^n is

$$\boxed{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T} \cdot \frac{\partial h}{\partial t}}$$

S = Storage Constant
T = Transmissibility

* The governing PDE for Unsteady, Saturated flow in confined aquifer
  - → Homogenous
  - → Isotropic

* Eq^n is also called Diffusion Eq^n

Governing Eq^{n}s for Unconfined AQuifer

* 2-D, Saturated flow — Unconfined

  * for Confined case
    - → Flow is purely horizontal
    - → streamlines / flow lines are parallel.

  * Unconfined
    - → The GWT represents a streamline
    - → Pressure @ GWT is atmospheric.

  → These boundary conditions cause difficulties to solve analytically.

→ Dupit (1863) Simplified the approach. → Assumptions

(i) The curvature of the free surface i.e GWT is small → The streamlines can be assumed to be horizontal at all sections.

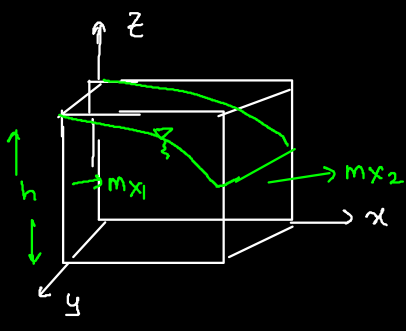
(2) The hydraulic gradient line = Slope of GWT & does not vary with depth.

$$\left(\frac{\partial h}{\partial z} = 0\right)$$

We consider the prism formed by the GWT

V_x = velocity in the x-direction

V_y = velocity in the Y-direction.



For steady flow. → RTT

$$\frac{d}{dt}\iiint_{CV} \beta\rho\, d\forall + \iint_{CS} \beta\rho \vec{v}\cdot d\vec{A} = 0 \quad \text{and } \beta = 1$$

$$\Rightarrow \iint_{CS} \beta\rho\vec{v}\cdot d\vec{A};$$

x dir^n:- $\boxed{m_{x_1} = \rho V_x\cdot(h\Delta y)}$ | mass influx in x dir^n. m_{x_1}

$$m_{x_2} = \rho V_x(h\Delta y) + \frac{\partial}{\partial x}(\rho V_x h \Delta y)\Delta x$$

Net outflux:- $mx_2 - mx_1$

$$\Rightarrow \boxed{\frac{\partial}{\partial x}(\rho V_x h)\Delta x\Delta y}$$

Similarly in Y-dir^n ⇒ $\boxed{\frac{\partial}{\partial y}(\rho V_y h)\Delta x\Delta y}$

⇒ From the continuity Eq^n ⇒ $\iint_{CS} \rho\vec{v}\, d\vec{A} = 0$

$$\Rightarrow \frac{\partial}{\partial x}(\rho V_x h)\Delta x\Delta y + \frac{\partial}{\partial y}(\rho V_y h)\Delta x\Delta y = 0$$

Use Darcy Law ⇒ $V_x = -k_x\frac{\partial h}{\partial x}$; $V_y = -k_y\frac{\partial h}{\partial y}$.

$$\Rightarrow \frac{\partial}{\partial x}\left(-k_x\frac{\partial h}{\partial x}\cdot\rho h\right) + \frac{\partial}{\partial y}\left(-k_y\cdot\frac{\partial h}{\partial y}\cdot\rho h\right) = 0$$

⇒ Assume $k_x = k_y$ (Isotropic) and Incompressible.

$$\Rightarrow k\rho\cdot\frac{\partial^2 h}{\partial x^2}\cdot h + k\rho\cdot\frac{\partial^2 h}{\partial y^2}\cdot h = 0$$

| $h\frac{\partial^2 h}{\partial x^2}$ → $\frac{1}{2}\cdot\frac{\partial^2 h^2}{\partial x^2}$

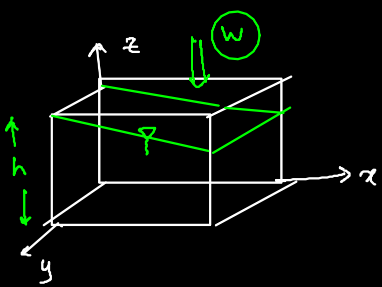
$$\Rightarrow \boxed{\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = 0} \qquad \boxed{\nabla^2 h^2 = 0}$$

Governing PDE for 2-D steady Incompressible flow → Unconfined

→ In unconfined aquifers, we may have inflow due to Recharge.

Ⓐ Unconfined aquifer with Recharge → The governing Eq^n remains the same, but an extra term is added.

① The x and y direction fluxes remain the same. There will be additional mass flux in the z-direction.



② Let Ⓦ be the incoming recharge ($m^3/s / m^2$ Area)

ΔM_z = Mass flux in Z-dir^n

$$\boxed{\Delta M_z = \rho w \Delta x \Delta y}$$

Now apply.

$$\iint_{c.s} \rho \vec{V}\cdot d\vec{A} = \frac{\partial}{\partial x}(\rho V_x h)\Delta x \Delta y + \frac{\partial}{\partial y}(\rho V_y h)\Delta x \Delta y + \rho w \Delta x \Delta y = 0$$

⇒ Combine this with Darcy's Law; Simplify.

⇒ $\boxed{k_x = k_y}$ and Incompressible.] Governing differential Eq^n → Steady → Isotropic

Unconfined, with Recharge (w)

$$\boxed{\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = -\frac{2w}{k}}$$

$$\boxed{\nabla^2 h^2 = -\frac{2w}{k}}$$

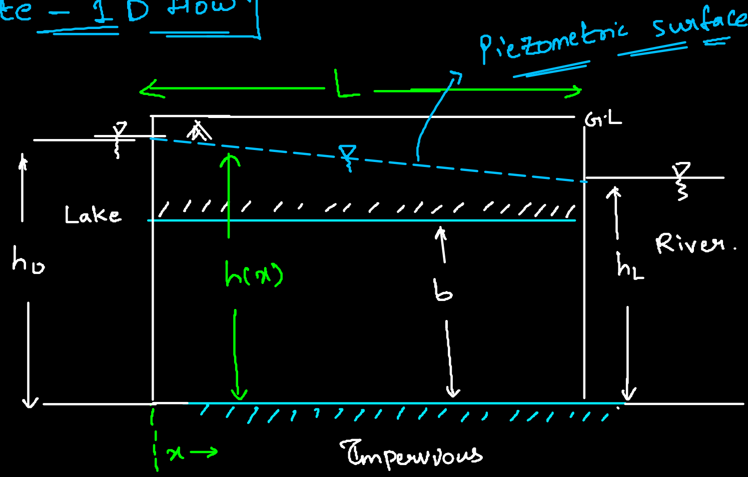
Simplified GW flow situations.

Ⓐ Confined Aquifer ⇒ Steady State – 1D flow.

→ The flow is Steady

→ Homogenous, Incompressible.

→ We have $\boxed{\nabla^2 h = 0}$



⇒ For 1D Case ⇒ $\boxed{\frac{\partial^2 h}{\partial x^2} = 0}$

BC's:- At $x = 0$; $h = h_0$

$x = L$, $h = h_L$

$$\Rightarrow \boxed{h = c_1 x + c_2} \rightarrow \text{Find } c_1 \text{ and } c_2$$

At $x = 0 \rightarrow h = h_o \Rightarrow \boxed{h_o = c_2}$

$x = L,\ h = h_L \Rightarrow h_L = c_1 L + h_o$

$$\Rightarrow \left(\frac{h_L - h_o}{L}\right) = c_1$$

$$\Rightarrow \boxed{h = h_o + \left(\frac{h_L - h_o}{L}\right) x}$$

↳ Linearly decreasing HGL

→ Discharge per unit width $\Rightarrow q = -k \cdot \dfrac{dh}{dx} \times (b \times 1)$

$$\Rightarrow \boxed{q = -kb \cdot \left(\frac{h_L - h_o}{L}\right)} \Rightarrow \boxed{q = Kb\left(\frac{h_o - h_L}{L}\right)}$$

↳ $q = m^3/s/m$ width

1-D flow → Unconfined aquifer with recharge

Dupit's Assumption

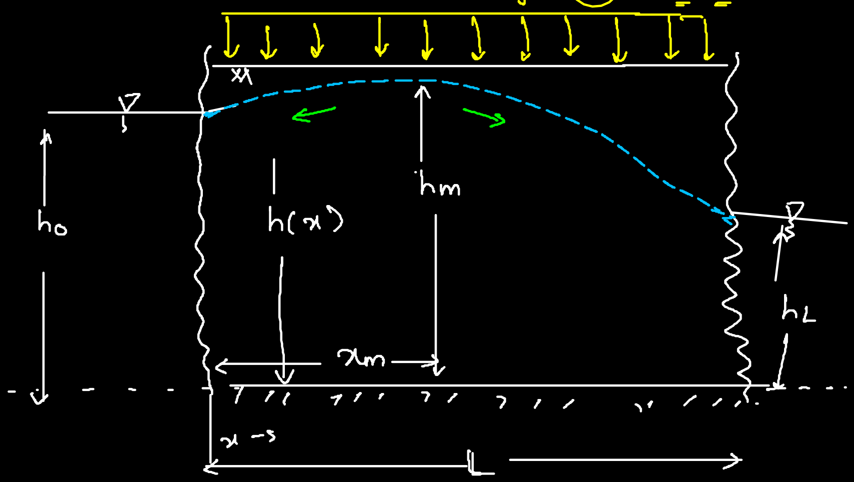
$$\frac{\partial^2 h^2}{\partial x^2} = \frac{-2w}{k}$$

B.C's are ⇒ $h = h_o$ @ $x = 0$

$h = h_L$ @ $x = L$.

$$\Rightarrow \boxed{h^2 = -\frac{W}{k} x^2 + c_1 x + c_2}$$

Uniform Recharge w (m/s)



Use the BC's

⇒ $\boxed{h_o = c_2}$ and

$h_L^2 = -\dfrac{W}{k} L^2 + c_1(L) + h_o$

$$\Rightarrow c_1 = -\frac{(h_o^2 - h_L^2 - WL^2/k)}{L}$$

$$\Rightarrow h^2 = -\frac{W}{k}x^2 - \frac{(h_o^2 - h_L^2 - WL^2/k)}{L}x + h_o$$

$$\Rightarrow \boxed{h^2 = -\frac{W}{k}x^2 - \frac{(h_o^2 - h_L^2 - WL^2/k)}{L}x + h_o}$$

↳ Solution of 1-D saturated flow in unconfined aquifer under Dupit's and Recharge

→ Equation of an ellipse → Rises initially and then falls

↳ Peak (hm) at (m)

④ Location of the maximum head ⇒ Water divide → Here the water flows in both directions at the water divide.

④ At the peak head ⇒ $\frac{dh}{dx} = 0$ at $x = x_m$ (or $\frac{\partial h^2}{\partial x^2} = 0$ at $x = x_m$

$$\Rightarrow \quad -\frac{2W}{K}x - \left(\frac{h_0^2 - h_L^2 - WL^2/K}{L}\right) = 0$$

$$\Rightarrow \quad \frac{2Wx_m}{K} = \frac{WL^2}{KL} + \frac{h_L^2}{L} - \frac{h_0^2}{L}$$

$$\Rightarrow \quad x_m = \frac{K}{2W}\left(\frac{WL}{K} + \frac{(h_L^2 - h_0^2)}{L}\right)$$

$$\Rightarrow \quad \boxed{x_m = \frac{L}{2} + \frac{K}{2WL}\left(h_L^2 - h_0^2\right)}$$

→ Then h_{max} ⇒ Substitute x_m in $h(x)$

⇒ Discharge per unit width

$$q_x = V_x \cdot (\text{Area}) = -K\frac{dh}{dx} \cdot (h \times 1)$$

$$= q_x = -Kh \cdot \frac{dh}{dx}$$ → Substitute from above.

$$\Rightarrow \quad \boxed{q_x = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_L^2\right)}$$

At $x = 0$

$$\Rightarrow \quad \boxed{q_0 = -\frac{WL}{2} + \frac{K}{2L}\left(h_0^2 - h_L^2\right)}$$

At $x = L$

$$\boxed{q_L = \frac{WL}{2} + \frac{K}{2L}\left(h_0^2 - h_L^2\right)}$$

$$\Rightarrow \quad \boxed{q_L = q_0 + WL}$$

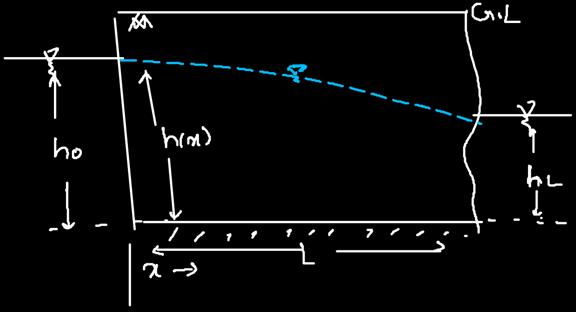
1-D saturated flow without Recharge. → Under Dupit's assumption

(A) Put $W=0$ in above Eq^{n}s.

$$\Rightarrow h^2 = -\frac{\cancel{W}}{K}x^2 - \frac{\left(h_L^2 - h_0^2 - \frac{\cancel{W}L^2}{K}\right)x}{L} + h_0^2$$

$$\Rightarrow \boxed{h^2 = h_0^2 - \frac{(h_0^2 - h_L^2)x}{L}}$$

⇐ Parabolic decreasing



⇒ The discharge (q_x) for unit width.

$$\Rightarrow 2h\cdot\frac{dh}{dx} = -\frac{(h_0^2 - h_L^2)}{L}$$

$$\Rightarrow h\cdot\frac{dh}{dx} = \frac{-(h_0^2 - h_L^2)}{2L} \Rightarrow q_x = K\cdot\frac{\partial h}{\partial x}\cdot(h\times 1)$$

$$\Rightarrow \boxed{q_x = \frac{-K}{2L}(h_L^2 - h_0^2)} \Rightarrow \boxed{q = \frac{K}{2L}(h_0^2 - h_L^2)}$$

(A) Analysis of underground drainage Structure:-

→ $h_0 \approx h_L \approx$ Negligible.

→ Flow is in ⊥ar to the screen.

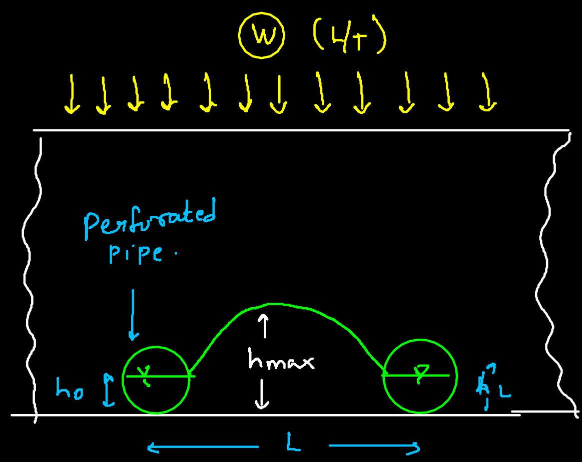
→ We have

$$h^2 = h_0^2 - \frac{W}{K}(x^2) - \frac{\left(h_0^2 - h_L^2 - \frac{WL^2}{K}\right)x}{L}$$

Put $h_0 \approx h_L \approx 0$

$$\Rightarrow h^2 = \frac{Wx^2}{K} + \frac{WL^2x}{KL}$$

$$\boxed{h^2 = \frac{W}{K}(L-x)x}$$



Location of h_{max} → $\frac{dh}{dx} = 0$

$\Rightarrow \boxed{x = L/2}$ → occurs at the midpoint

$$h_{max} = \sqrt{\frac{W}{K}\left(L - \frac{L}{2}\right)\frac{L}{2}} = \boxed{\sqrt{\frac{WL^2}{2K}}}$$

(A) Discharge:-

$$q_x = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}(\cancel{h_0^2 - h_L^2})$$

$$\boxed{q_x = W\left(x - \frac{L}{2}\right)}$$

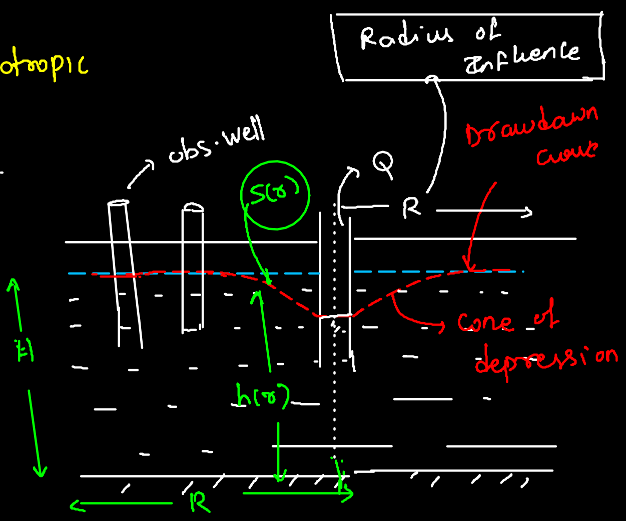
At $x=0$, $\boxed{q = -\frac{WL}{2}}$ ← Towards left

$x = L$ $\boxed{q = \frac{WL}{2}}$ → Towards right

⇒ Total $\boxed{q = WL}$ → Depends on the recharge rate

Well Hydraulics

④ Unconfined aquifer:- ④ Homogenous and Isotropic



S = Drawdown → S(r) ; Q = Pumping discharge.

h = height of GWL at radius (r)

H → Height of GWL at r = R (far away)

④ Drawdown:- Drop in the groundwater table from the original static level, when the water is pumped.

④ Cone of depression:- Shape of the 3D transition Surface as the water is being pumped.

④ Area of Influence:- It is the areal extent of the cone of depression.

* Beyond area of influence ⇒ drawdown = 0

④ Initially, the flow will be unsteady ⇒ h = f(r,t) and S = f(r,t).

④ The pumped water comes out from the storage in the aquifer.

④ With Prolonged Pumping at the Same rate ⇒ Equilibrium state is reached b/w the rate of pumping and the rate of inflow of GW into the aquifer; from the edges.

④ Under steady state ⇒ $\boxed{h = h(r) \text{ and } S = S(r)}$

→ The cone of depression remains constant with time. → Equilibrium condition

④ When the pumping is stopped ⇒ The cone of depression fills up until it reaches original level.

↳ No outflow, but still there is inflow ⇒ $\boxed{\text{Recuperation}}$ ⇒ $\boxed{\text{Unsteady}}$

④ The recuperation time depends on the aquifer characteristics.

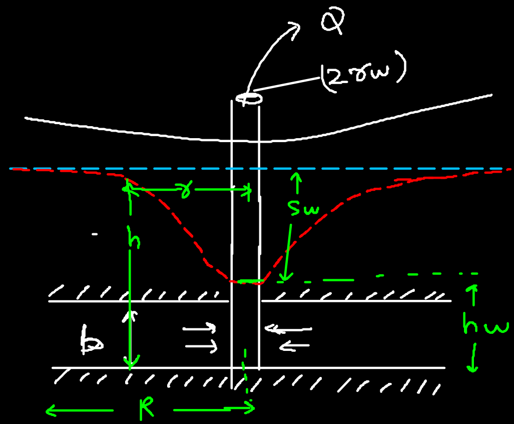
④ For confined aquifer ⇐ $\boxed{\text{Same process applies}}$ → But piezometric levels form the Cone of influence

④ $\boxed{\text{Recovery in a Confined aquifer is very quick compared to unconfined aquifer}}$

↳ B/c of pressure In the confined aquifer.

① Steady Radial flow → Well — Confined Aquifer

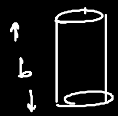
* b = Aquifer thickness, Q = Pumping Rate
* h = height of piezometric surface @ r
* h_w = " " " " @ well → r_w
* S_w = drawdown at the well



→ Apply Darcy Law

* V_r = Radial velocity

⇒ $V_r = K \cdot \left(\frac{dh}{dr}\right)$, In a cylinder of area $= (2\pi r)b$



⇒ Discharge; $Q = V_r \cdot A$ ⇒ $Q = (2\pi r)b \cdot K \frac{dh}{dr}$

⇒ $\boxed{Q = 2\pi T r\left(\frac{dh}{dr}\right)}$

⇒ $\int_{r_1}^{r_2} \frac{Q}{2\pi T} \cdot \frac{dr}{r} = \int_{h_1}^{h_2} dh$

Thiem's Eqⁿ

↓ Equilibrium Eqⁿ

Steady state Discharge.

⇒ $\frac{Q}{2\pi T} \ln\left(\frac{r_2}{r_1}\right) = h_2 - h_1$. ⇒ $\boxed{Q = \frac{2\pi T(h_2 - h_1)}{\ln(r_2/r_1)}}$

Ⓐ Generally, we replace 'h' with S. ⇒ $h_1 = H - S_1$ and $h_2 = H - S_2$

⇒ $Q = \frac{2\pi T(H - S_2 - H + S_1)}{\ln(r_2/r_1)}$ ⇒ $\boxed{Q = \frac{2\pi T(S_1 - S_2)}{\ln(r_2/r_1)}}$

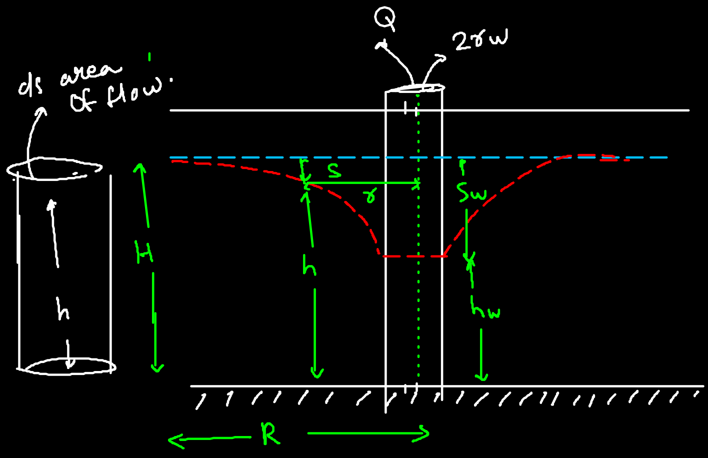
Ⓐ When $r_1 = r_w$ (at the well) ⇒ $S_1 = S_w$ and $h_1 = h_w$.

$r_2 = R$ (Radius of influence) ⇒ $S_2 = 0$ and $h_2 = H$

⇒ $\boxed{Q = \frac{2\pi T(S_w)}{\ln(R/r_w)}}$

Ⓐ T = transmissibility can be found using this equation.

Ⓐ Steady Radial flow into well : Unconfined



Ⓐ Use Dupit's assumptions.

Ⓐ Apply Darcy's Law : $V_r = K\dfrac{dh}{dr}$

→ Discharge, $Q = V_r \cdot$ Area.

$\Rightarrow Q = V_r \cdot (2\pi r)\cdot h$

$\Rightarrow Q = K\cdot\dfrac{dh}{dr}\cdot 2\pi r h$

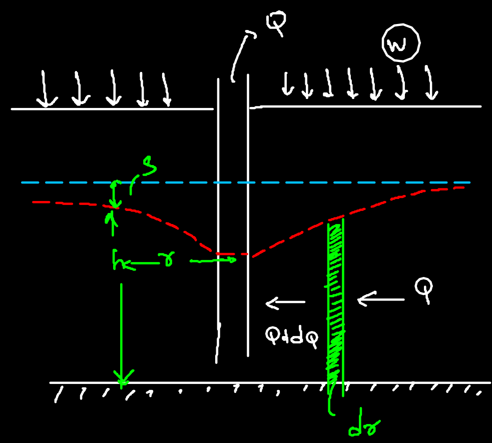
$\Rightarrow \dfrac{Q}{2\pi K}\cdot\dfrac{dr}{r} = h\,dh \Rightarrow \displaystyle\int_{r_1}^{r_2}\frac{Q}{2\pi K}\frac{dr}{r} = \int_{h_1}^{h_2} h\,dh$

$$\Rightarrow \boxed{Q = \frac{\pi K(h_2^2 - h_1^2)}{\ln(r_2/r_1)}}$$

Ⓐ when $r = R \Rightarrow h = H$
$r = r_w, h = h_w$

$$\boxed{Q = \frac{\pi K(H^2 - h_w^2)}{\ln(H/h_w)}}$$

Well in Unconfined Aquifer with Recharge



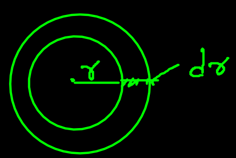
Ⓐ If there is no recharge → Then $Q = Q_{inflow}$. But now, we have an additional influx into the aquifer.

Q at the well = highest

Q_{in} in the aquifer = $f(r)$.

→ As one approaches the well, Q increases

→ Consider the cylindrical cell as shown.



→ Let dQ be the increase in the flow in the aquifer at some r due to incoming recharge.

$\Rightarrow dQ =$ Velocity × Area
$= -W \times (2\pi r)\,dr$

$$\Rightarrow \boxed{dQ = -2\pi r W\,dr}$$

(-ve indicates as r increases, Q decreases)

$\Rightarrow$ Integrating

$$\Rightarrow \quad Q = -2\pi W \frac{r^2}{2} + c$$

⊛ Use the BC's as At the well $\Rightarrow r = 0, \; Q = Q_w \Rightarrow C = Q_w$

$$\Rightarrow Q = Q_w - 2\pi W \frac{r^2}{2}$$

$$\Rightarrow \boxed{Q_r = Q_w - \pi W r^2}$$ ← Flow in unconfined aquifer at any radius (r).

$\Rightarrow$ For unconfined aquifer $\Rightarrow Q = -2\pi r K h \frac{dh}{dr}$

$$\Rightarrow \quad -\pi r^2 W + Q_w = -2\pi r K h \frac{dh}{dr}$$

$$\Rightarrow \quad \frac{-\pi r^2 W + Q_w}{r} \cdot dr = -2\pi K h \cdot dh$$

$$\Rightarrow \int_{r_1}^{r_2} \left(\frac{Q_w}{r} - \pi r W \right) dr = -2\pi K \int_{h_1}^{h_2} h\,dh$$

$$\Rightarrow \quad Q_w \cdot \ln\left(\frac{r_2}{r_1}\right) - \pi W \cdot \frac{(r_2^2 - r_1^2)}{2} = -2\pi K \frac{(h_2^2 - h_1^2)}{2}$$

Use BC's

$$\Rightarrow \boxed{H^2 - h^2 = \frac{W}{2K}(r^2 - R^2) + \frac{Q_w}{\pi K} \ln(R/r)}$$

$\Rightarrow$ At $r = R \Rightarrow \boxed{Q = 0}$

$$\Rightarrow \quad 0 = Q_w - \pi W R^2$$

$$\Rightarrow \boxed{Q_w = \pi W R^2}$$ ← Discharge at the well

---

Unsteady flow → Confined Aquifer

Ⓐ The governing Eq^n is

$$\boxed{\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial h}{\partial r} = \frac{S}{T} \cdot \frac{\partial h}{\partial t}}$$

Theis, proposed a solution to this Eq^n with the BC's :- $h = H$ @ $t = 0$; $h \to H$, $r \to \infty$ for $t \geq 0$

$$\boxed{s = (H-h) = \frac{Q}{4\pi T}\int_u^{\infty} \frac{e^{-u}}{u}\, du}$$ ← Non-equilibrium equation

$$\boxed{s = \frac{Q}{4\pi T} W(u)}$$

$W(u) \Rightarrow$ well function $= \int_u^{\infty} \frac{e^{-u}}{u}\, du$

→ $u = f(t, r) \Rightarrow \boxed{u = \frac{r^2 S}{4Tt}}$

$$\Rightarrow W(u) = -0.577216 - \ln(u) + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} + \cdots$$

Assumptions for Theis solution

(1) Aquifer is homogenous, isotropic and of uniform thickness and infinite areal extent.

(2) Before pumping, the piezometric surface is horizontal.

(3) The pumping rate is constant

(4) The well penetrates the confined aquifer completely → flow lines are horizontal

(5) The well diameter is so small that the storage within the well can be neglected.

(6) The water removed from storage is discharged instantaneously.

→ Theis' Solution for determination of Aquifer parameters.

→ Graphical Method $\boxed{\text{Pumping tests}}$

$$\boxed{s = \frac{Q}{4\pi T} W(u)} \quad \text{and} \quad u = \frac{r^2 S}{4Tt} \Rightarrow \boxed{\frac{r^2}{t} = \left(\frac{4T}{S}\right) u}$$

Both look similar.

→ The relaⁿ b/w s and W(u) is similar to that b/w $\frac{r^2}{t}$ and u

↳ This was used to develop the graphical solution.

Procedure:-

(1) Prepare a log-log plot b/w $\boxed{W(u)}$ (y-axis) and (u) (x axis) ⇒ $\boxed{\text{Type curve}}$ → Available in standard tables

(2) Prepare a log-log plot b/w s (y axis) and $\left(\frac{r^2}{t}\right)$ (x-axis) using the same scale as above

③ The observed (s) v/s (r^2/t) curve is superimposed on the type-curve.

↳ By keeping the co-ordinate axis parallel.

④ The two curves are then adjusted till a position is found such that most of the plotted points of (s v/s r^2/t) fall on the segment of the type curve.

⑤ With this matching, a convenient point is selected and the co-ordinate values of $W(u)$, (u), (s) and (r^2/t) are recorded.

⑥ The values of S and T are calculated using the known Eq^{n}s

$$\boxed{T = \frac{Q}{4\pi s} W(u)} \quad \text{and} \quad \boxed{S = \frac{4T}{(r^2/t)} u}$$

⊛ Cooper- Jacob method → we know that $u = \dfrac{r^2 S}{4Tt}$

* for small 'r' and large 't' ⇒ $u \approx$ small
* for Small $u \le 0.01$ ⇒ The first two terms of the $W(u)$ expansion are enough.

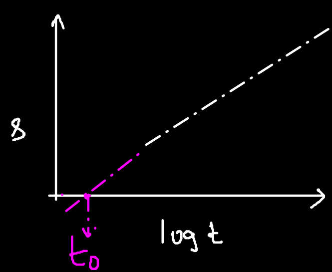
$$\Rightarrow \quad W(u) = \left[-0.5772 - \ln u\right]$$

$$\Rightarrow \quad W(u) = \left(-0.5772 - \ln \frac{r^2 S}{4Tt}\right)$$

⇒ We can get drawdown (s) as

$$\boxed{s = \frac{Q}{4\pi T}\left(-0.5772 - \ln \frac{r^2 S}{4Tt}\right)}$$

→ Plot s v/s $\log t$ ⇒ appear as a straight line



At $t = t_0$, $s = 0$

$$\Rightarrow \quad \frac{2.303\,Q}{4\pi T} \log \overbrace{\left(\frac{2.25\,T t_0}{r^2 S}\right)}^{\text{zero}} = 0$$

$$\Rightarrow \quad \frac{2.25\,T t_0}{r^2 S} = 1 \quad \Rightarrow \quad \boxed{S = \frac{2.25\,T t_0}{r^2}}$$

→ But how to get (T) ?

→ Next Eq^n ↓.

④ Select 2 drawdowns s_1 and s_2 so that $(t_2/t_1) = 10$

$$\Rightarrow s_1 = \frac{2.303Q}{4\pi T}\log_{10}\left(\frac{2.25Tt_1}{r^2 S}\right)$$

$$\Rightarrow s_2 = \frac{2.303Q}{4\pi T}\log_{10}\left(\frac{2.25Tt_2}{r^2 S}\right)$$

→ Substract

$$\Rightarrow \Delta S = S_2 - S_1 = \frac{2.303Q}{4\pi T}\underbrace{\log\left(t_2/t_1\right)}_{1}$$

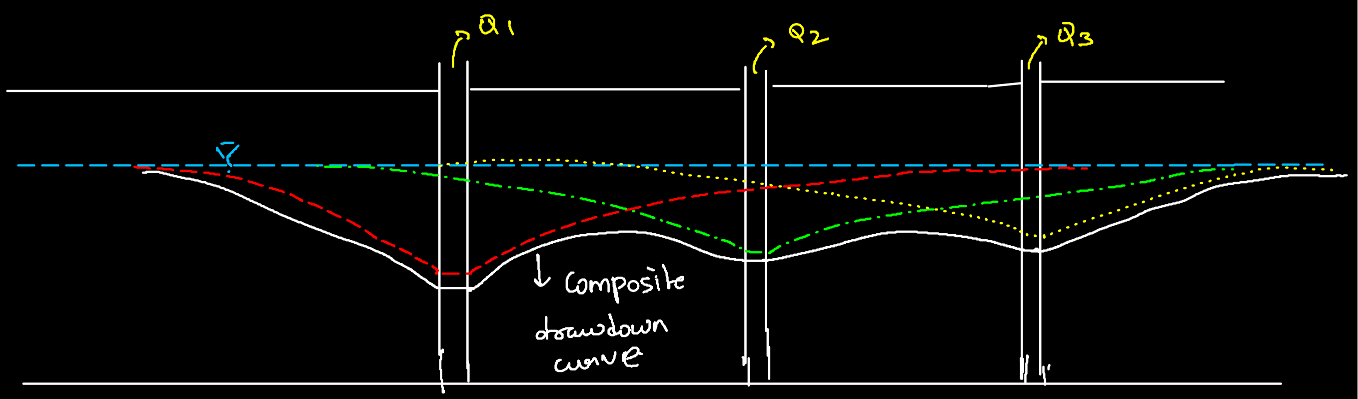
$(t_2/t_1) \rightarrow 10$

$$\rightarrow \Delta S = \frac{2.303Q}{4\pi T} \Rightarrow \boxed{T = \frac{2.303Q}{4\pi\,\Delta S}}$$

Multiple well systems

→ When two pumping wells are located close to each other, → The area of influence interfer.

→ We use the Method of superposition



$$\left.\begin{aligned} s_T^1 &= s_{11} + s_{12} + s_{13} \\ s_T^2 &= s_{21} + s_{22} + s_{23} \\ s_T^3 &= s_{31} + s_{32} + s_{33} \end{aligned}\right\} \Rightarrow \boxed{s_T^i = \sum_{j=1}^{n} s_{ij}}$$

→. The solutions for steady state and unsteady state are applicable.

→ Use:- → The wells for water supply should be as far as possible.

→ Least cost pumping well network may be designed.

→ Drainage wells, it is desirable to maximize the interference to control the GWT at construction sites